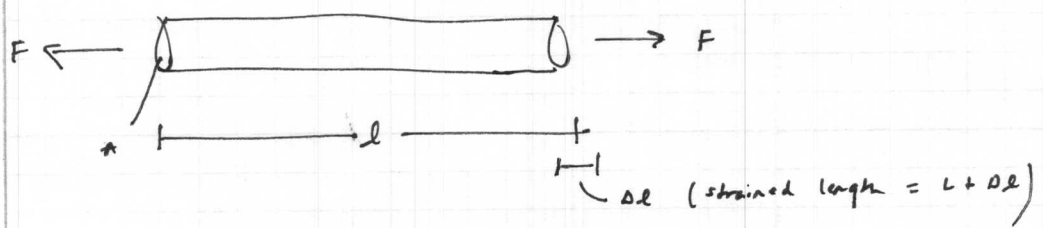
in a macroscopic solid, we have simple linear-elastic behavior

e.g. a bar in uniaxial tension

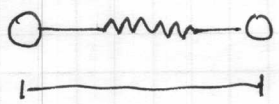


$$\sigma = \frac{F}{A} = E\varepsilon$$ ← young's modulus [N/m²]

$$\varepsilon = \frac{\Delta l}{l}$$

$$\frac{F}{A} = E\frac{\Delta l}{l} \Rightarrow \Delta l = \frac{FL}{AE}$$ ~ we'll return here later.

at a far smaller scale, let's consider a material as a massive array of springs

—o each bond is like a spring 

this is modeled by an interatomic potential, $x_0 = r_e$ ← equilibrium separation

e.g., the morse potential

$$U = D\left[\left(1 - e^{-\beta(r-r_0)}\right)^2 - 1\right]$$

D ↑ well "depth"

β ← well width.

$U(x)$ = energy [J]

$\frac{dU}{dx}$ = force [N]

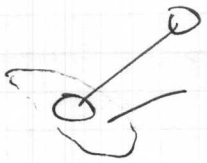
$\frac{d^2U}{dx^2}$ = stiffness [N/m]

thus, if our "material" is a single bond spring

the restoring force $F = s(x - x_0)$

s: stiffness, x_0: equilibrium length.

define "area" $= \frac{1}{x_0^2}$



then stress $= \frac{F}{A} = \frac{F}{x_0^2} = \frac{s(x - x_0)}{x_0^2} = \frac{s}{x_0}\left(\frac{x - x_0}{x_0}\right)$

strain $= \frac{\Delta l}{l} = \varepsilon$

modulus $\left[N/m^2\right] = E$

let's approximate some real values.

based on interatomic potentials.

- covalent bonds, $s = 20 - 200$ N/m
- metal/ionic bonds, $s = 15 - 100$ N/m
- polymers, $s = 0.5 - 1$ N/m

} c-c bonds dominate springs in series

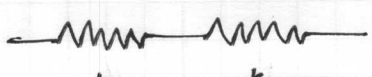
(c-c bonds are weak)

thus, if $E \approx \frac{s}{x_0}$

$x_0 = 0.2$ nm

$s = 200$ N/m.

$E \approx 1000$ GPa = graphite!



$k_{eff} = \left(\frac{k_1 + k_2}{k_1 k_2}\right)^{-1}$

if $k_1 << k_2 \Rightarrow k_{eff} = k_1$

stiffness can typically be close to the theoretical limit

strength, no... it is governed by defects, slip (e.g. grain boundaries)

and can be as low as $10^{-4}E$; in nanoscopic materials, we can reach theoretical strength limits.

strength, let's say bonds break @ 10% elongation.

$$\sigma = \frac{S}{x_0}\left(\frac{x - x_0}{x_0}\right) \Rightarrow \sigma \approx 0.1\,E$$

$\left(\frac{x - x_0}{x_0}\right) = 0.1$

→ see Ashby plot.

in polymer fibers, chain alignment is very high ⇒ we approach theoretical stiffness, but are away from strength limit, which is practically governed by chain - chain gaps.

defects are stress concentrators.