

# 06: Thermal properties of nanostructures

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**John Hart, Aaron Schmidt**

[ajohnh@umich.edu](mailto:ajohnh@umich.edu)

<http://www.umich.edu/~ajohnh>

# Announcements

- PS1 due **Wed at the beginning of lecture (10.40)**



# Recap: mechanical properties of nanostructures

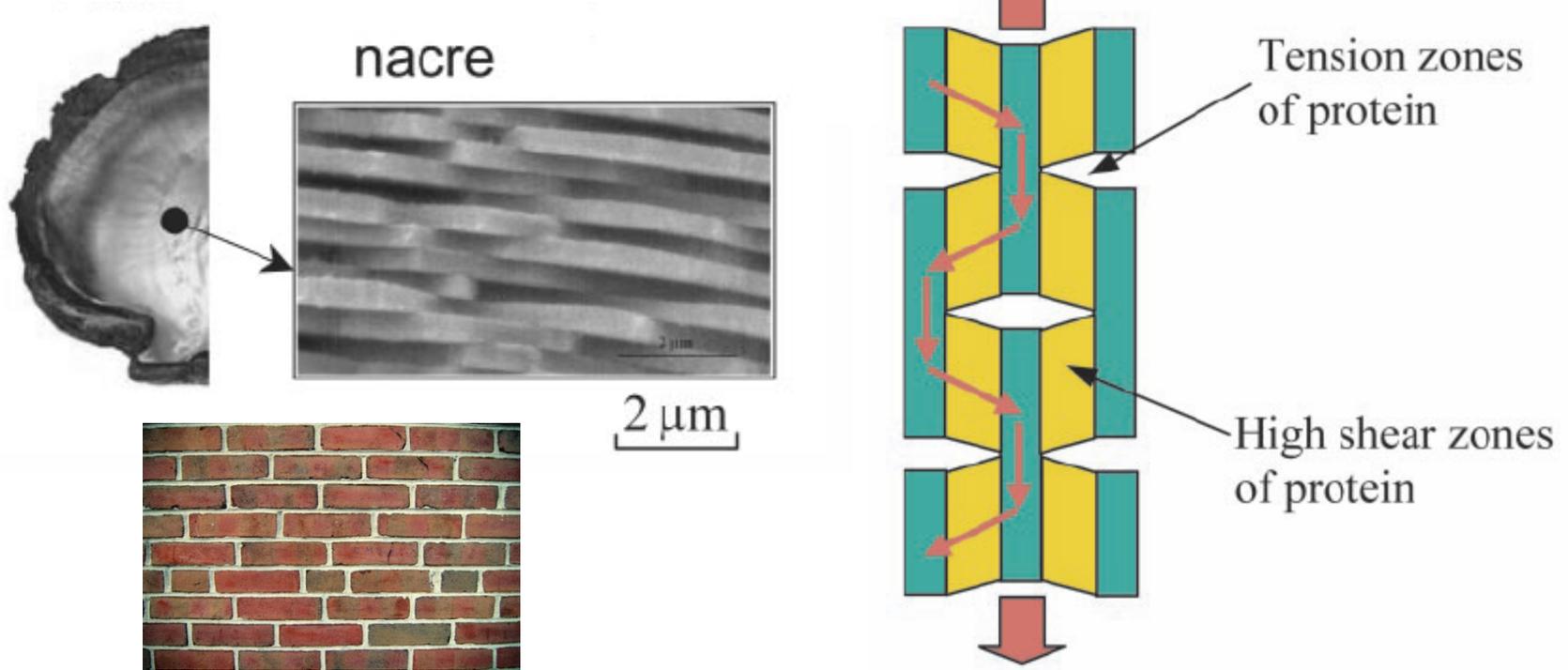


- Bonding determines mechanical properties
  - Simple spring model (potential well) predicts ultimate stiffness and strength
  - Ultimate stiffness is close to real stiffness of materials
  - Ultimate strength realized only in nanoscale volumes which can be defect free; due to discrete numbers of defects we see distributions of strengths in nanostructures
- “Top-down” engineering of material structure, e.g., grain size reduction can affect strength
- CNTs (and other wires/tubes) can be modeled as continuous beams
  - Must carefully assign property and geometric parameter values
  - Must consider “real” load-bearing area

# Nacre (mother of pearl): distributed flaw tolerance



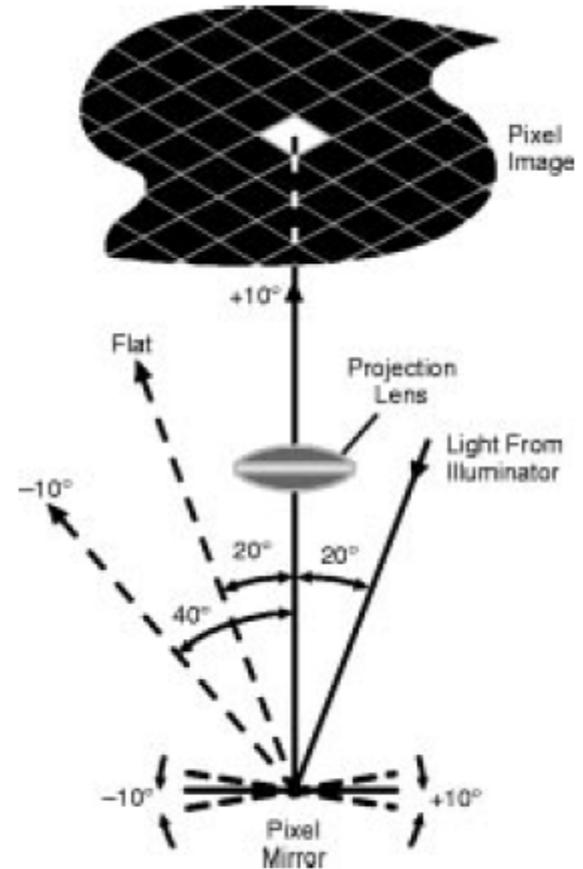
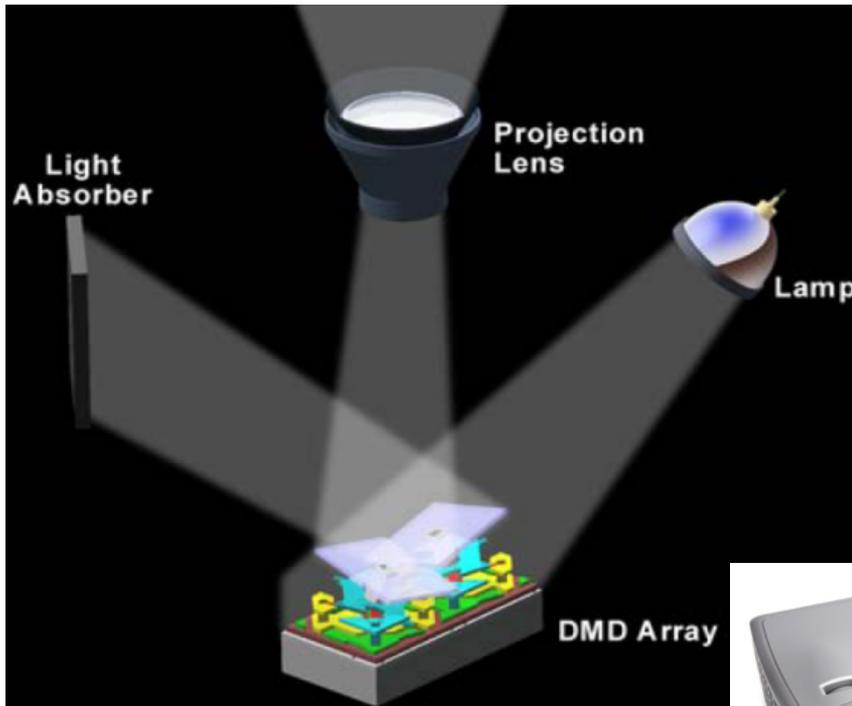
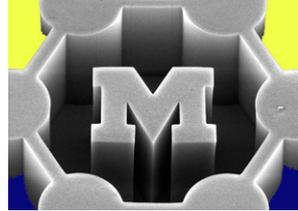
- Hexagonal platelets of aragonite (a form of calcium carbonate) 10-20  $\mu\text{m}$  wide and 0.5  $\mu\text{m}$  thick, arranged in a continuous parallel lamina, separated by sheets of elastic biopolymers (such as chitin, lustrin and silk-like proteins)



- Many other examples in nature (e.g., skeletons, snail shells)

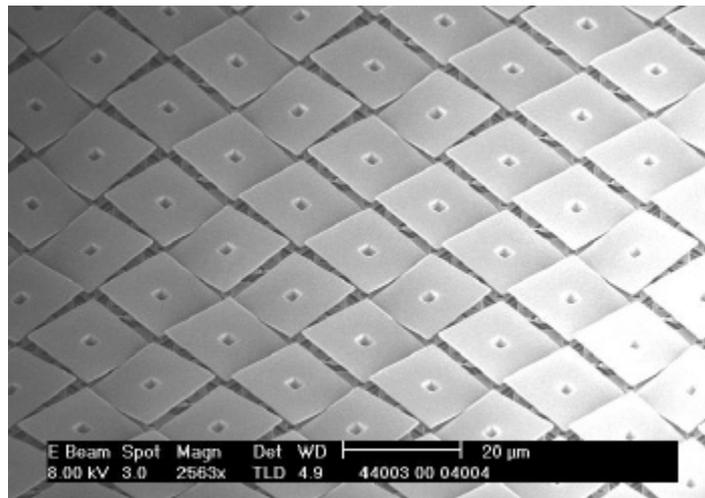
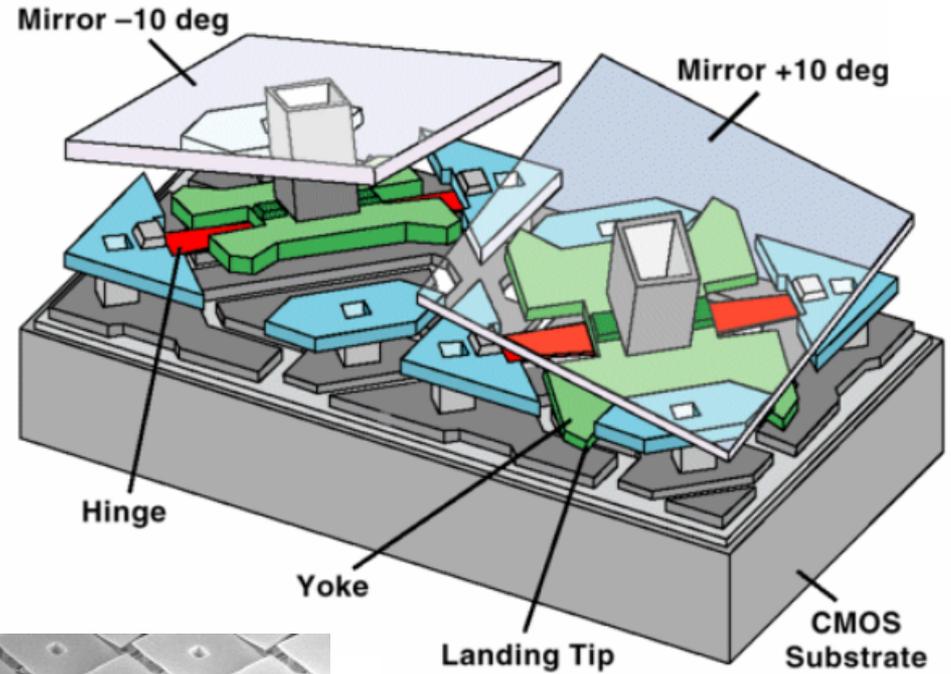
# DMD projector

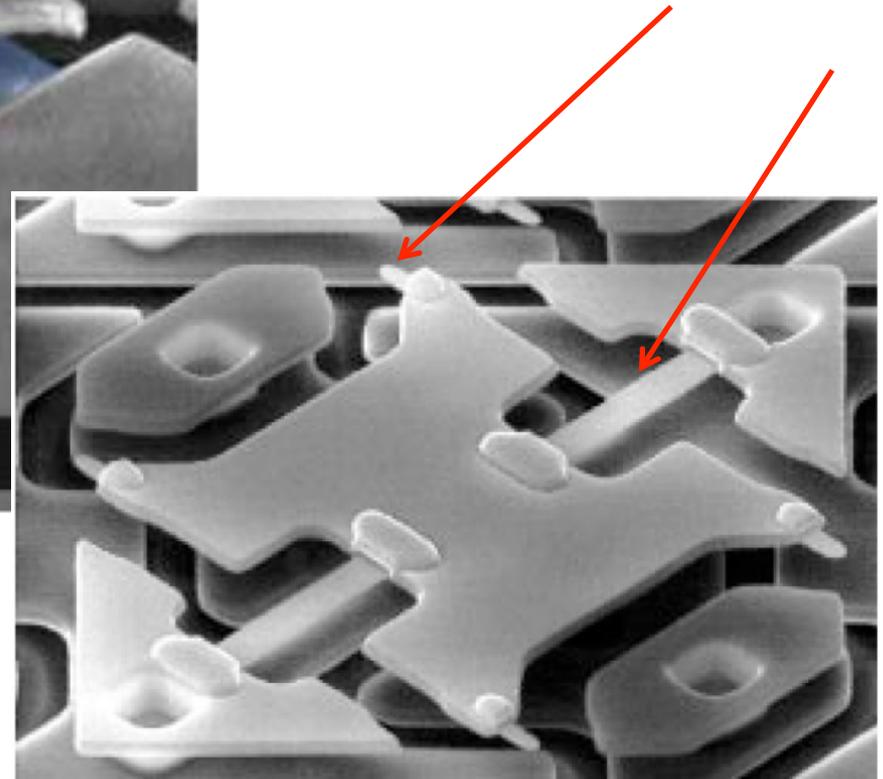
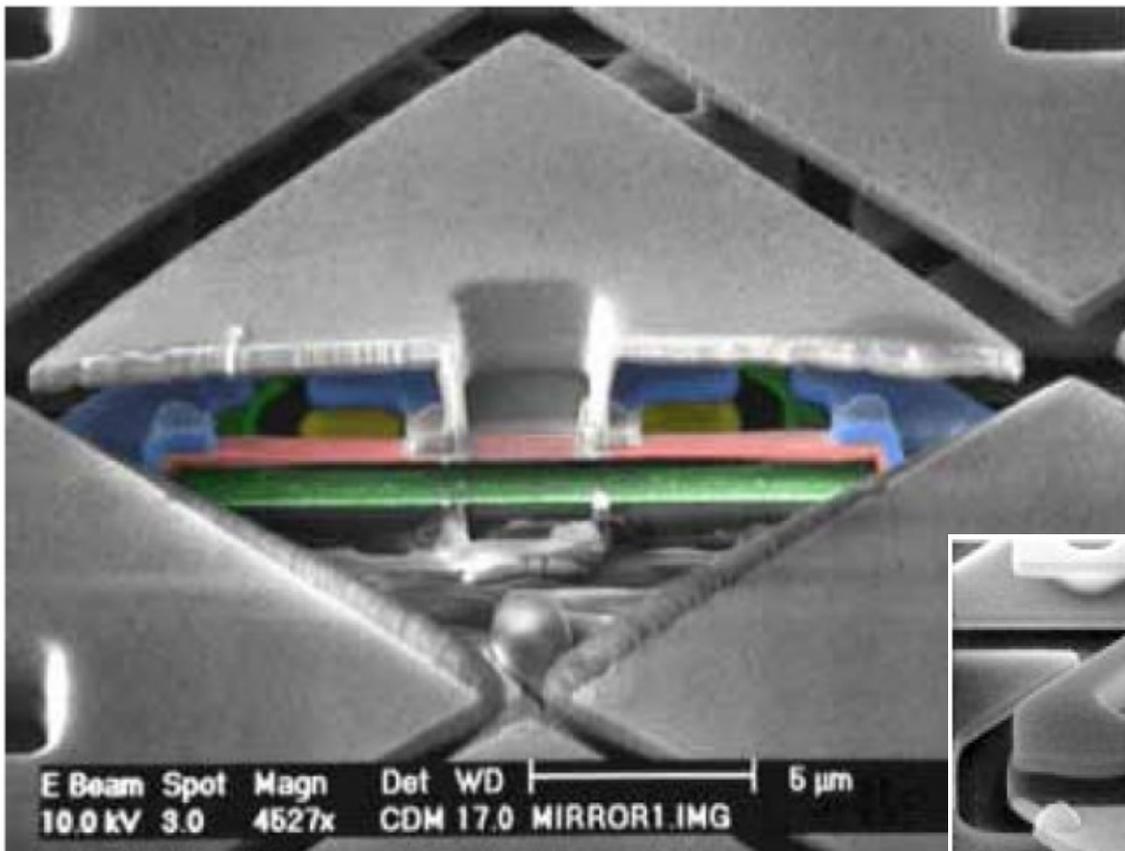
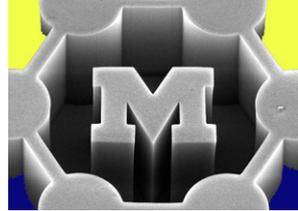
- Projection display based on changing the angle of micromirrors (electrostatic actuation)
- Invented 1987; shipped 1996.
- Support cantilevers are single-crystal Ni



# DMD structure

- 4 x 4 micron mirror
- Hinge (flexure)
- Yoke
- Connecting posts
- Electrostatic actuator
- CMOS control / addressing circuit (memory array)
- Moving parts are aluminum



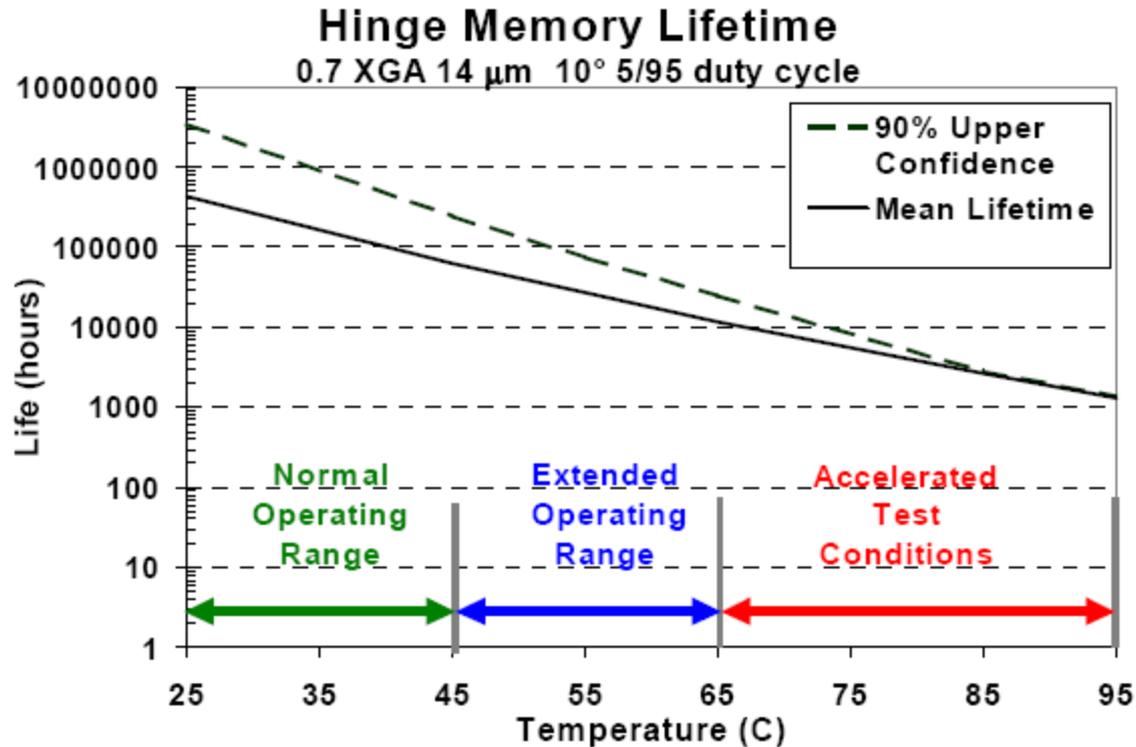


- The hinges are 60 nm thick by 600 nm wide
- The hinges are flexed  $\pm 10^\circ$
- For a bulk hinge, torsion will result in plastic deformation after only a few cycles

# Hinge fatigue and “memory”



- NO fatigue failures
  - $3 \times 10^{12}$  device cycles = 120 years life at 1000 hours per year
  - 500,000 mirrors per device  $\rightarrow$   $14 \times 10^{18}$  individual mirror cycles without a single hinge fatigue failure!



# Today's agenda

- Diffusive and ballistic thermal transport; quantum limit
- Measurements of nanoscale thermal properties: wires, nanotubes, and molecules
- Thermal interfaces
- Thermoelectric materials
- Near-field thermal radiation



# Today's readings (ctools)



## Nominal: (on ctools)

- Rogers, Pennathur, Adams, excerpt on Nanoscale Heat Transfer from Nanotechnology: Understanding Small Systems
- Kim et al., “Thermal transport measurements of individual multiwalled nanotubes”

## Extras: (on ctools)

- Cahill et al., “Thermometry and thermal transport in micro/nanoscale solid-state devices and structures”
- Shi et al., “Measuring thermal and thermoelectric properties of one-dimensional nanostructures using a microfabricated device”
- Schwab et al., “Measurement of the quantum of thermal conductance” (with news/views by Kouwenhoven)

# Heat and temperature



- **Heat** = random (thermal) energy. Electrons, phonons, photons and atoms can all carry heat
  - **Temperature** = a measure of average energy in each degree of freedom in a system
  - Example: ideal gas,  $\langle E \rangle$  of each degree of freedom =  $\frac{1}{2} k_B T$
- *Need many particles near equilibrium for temperature to have meaning*

# Modes of heat transfer



## ***Conduction***

- By motion of phonons within a solid (metals, semiconductors, insulators)
- By motion of electrons within a solid (metals, semiconductors)

## **Convection**

- By motion of molecules within a fluid

## **Radiation**

- By electromagnetic waves traveling through space, from one body to another (atoms not in contact)

# Diffusive thermal transport

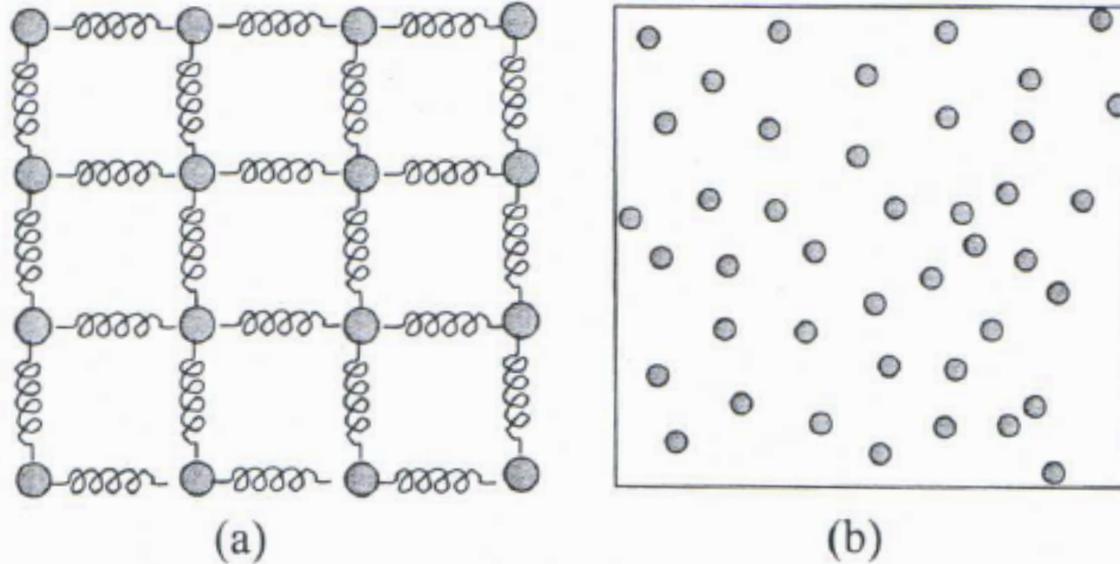


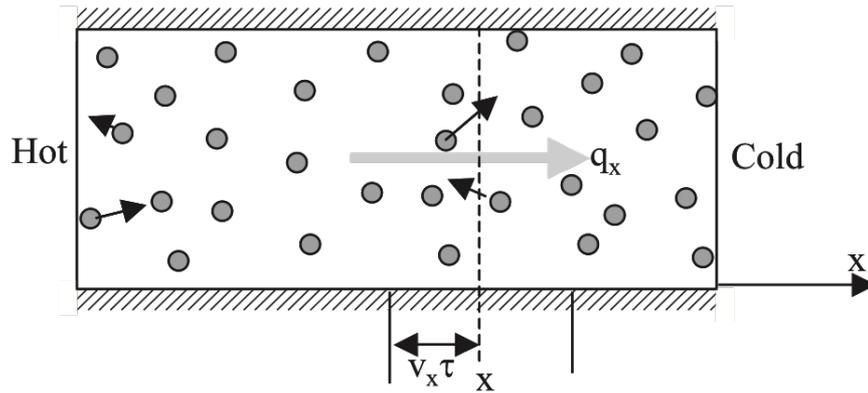
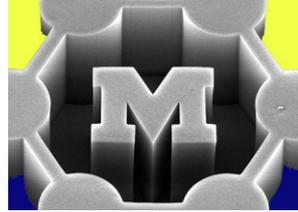
Figure 1.11 (a) A mass-spring system representing interconnected atoms in a crystal, and (b) phonon gas model replaces the solid atoms in a crystal.

- With enough collisions, "random walk" produces a diffusion process

$$q = -k\nabla T$$

Fourier's Law

# Diffusive thermal transport



$$q_x = \frac{1}{2}(nE v_x)|_{x-v_x\tau} - \frac{1}{2}(nE v_x)|_{x+v_x\tau}$$

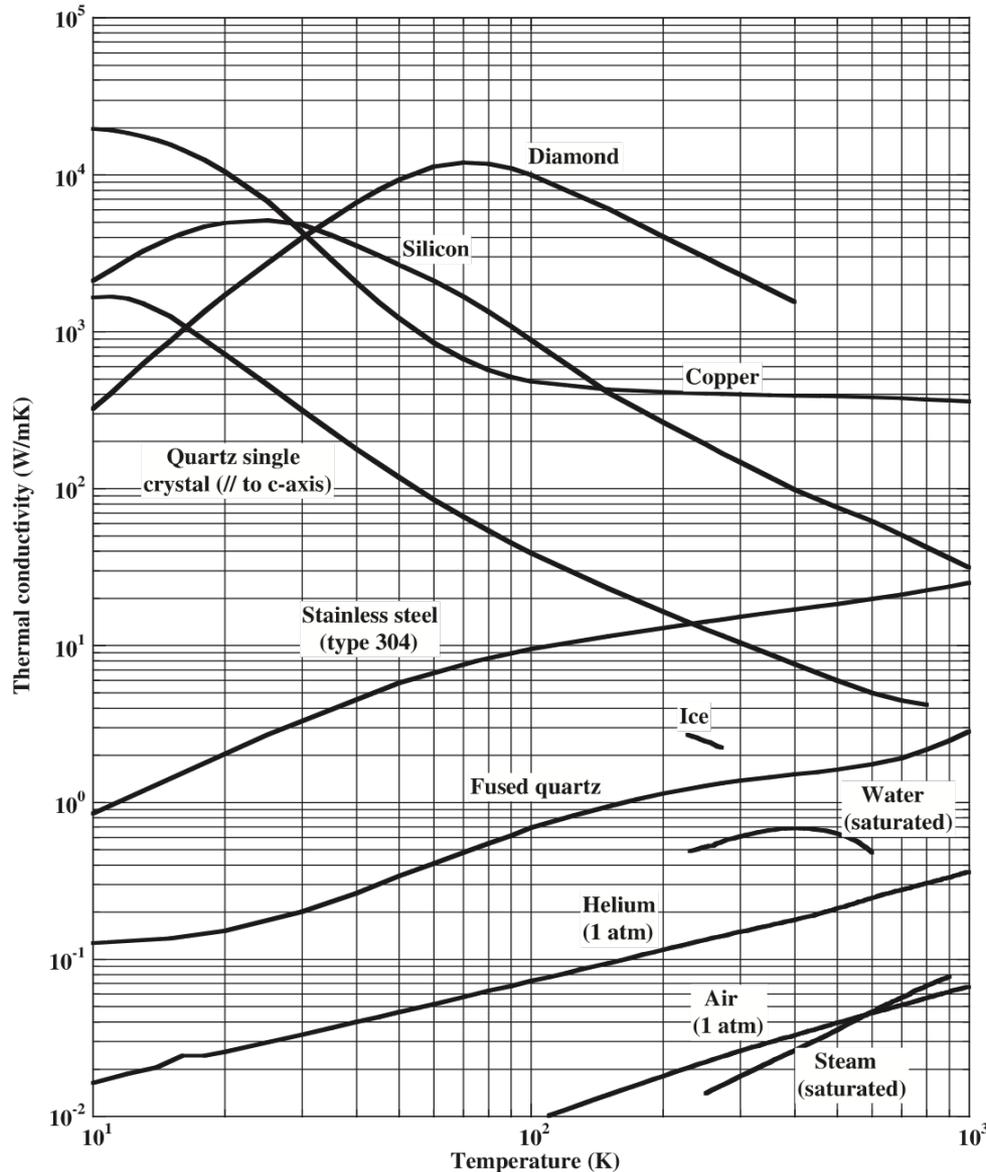
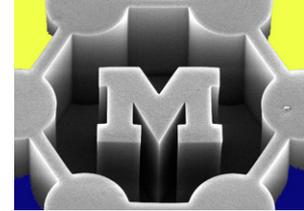
$$q_x = -v_x \tau \frac{d(nE v_x)}{dx}$$

$$U = nE \quad \frac{dU}{dT} = C$$

$$q_x = -\frac{v^2 \tau}{3} \frac{dU}{dT} \frac{dT}{dx}$$

$$q_x = -C v^2 \tau / 3 * dT / dx = -k dT / dx \longrightarrow k = C v^2 \tau / 3 = C v \Lambda / 3 = \rho c v \Lambda / 3$$

# Thermal conductivity of materials

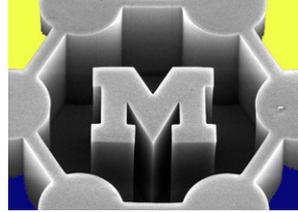


## Carrier contributions (at 300K)

	Phonons [W/mK]	Total [W/mK]
Diamond	2000	2000
Silicon	148	148
Copper	10	400
Platinum	4	71

Figure 1.5 Thermal conductivity as a function of temperature for representative materials (data from Touloukian et al., 1970, and <http://www.chrismanual.com/Default.htm>).

# When does Fourier's law fail?



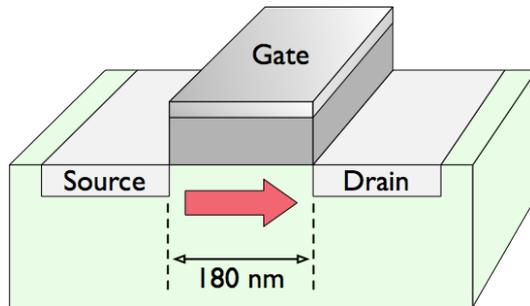
- When the carrier mean free path is comparable to the system size (i.e.,  $< 1 \mu\text{m}$ )
- At times less than carrier collision times (fast processes, i.e.,  $\ll 1 \text{ ns}$ )



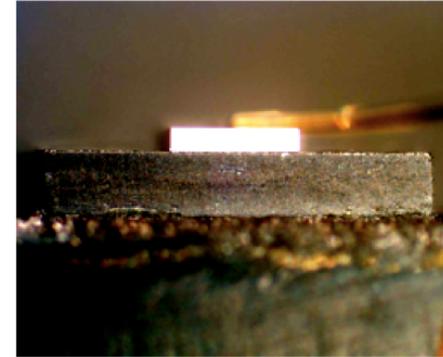
# Some examples



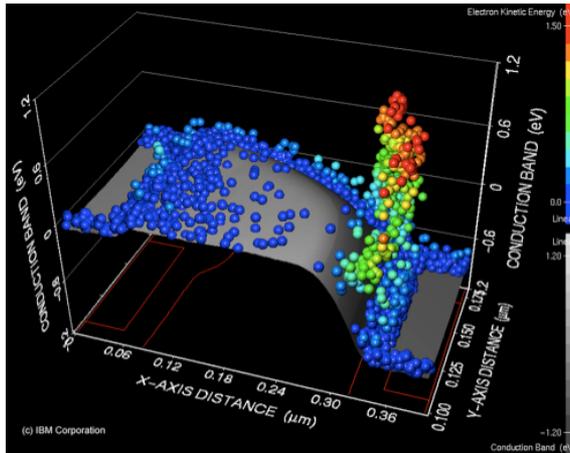
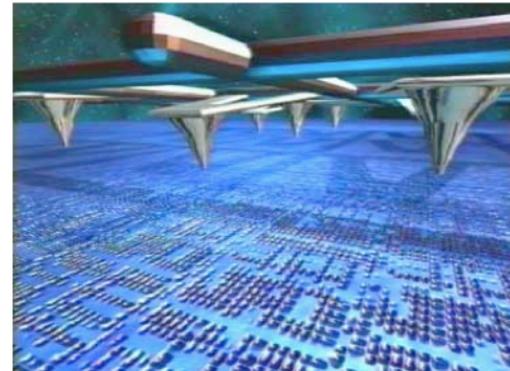
## Transistors



## Semiconductor lasers



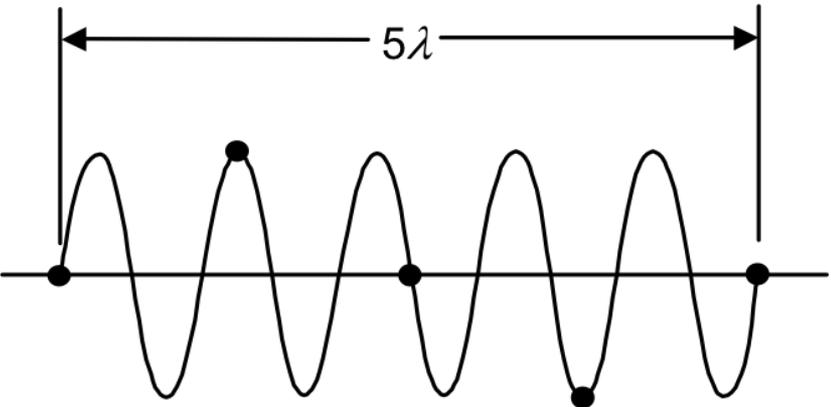
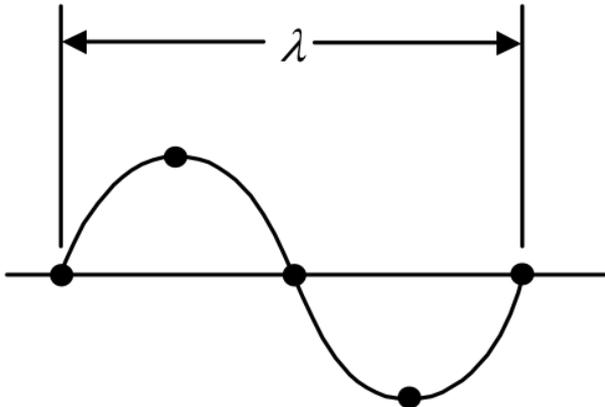
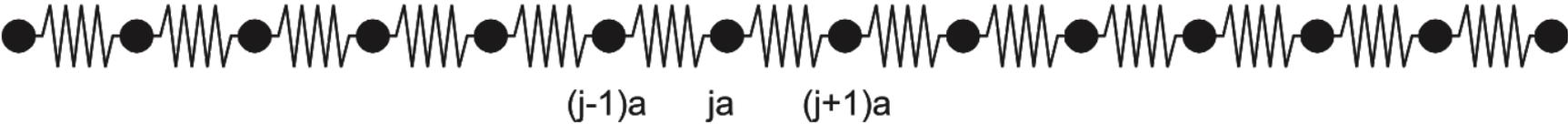
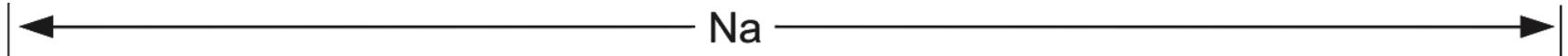
## Data storage



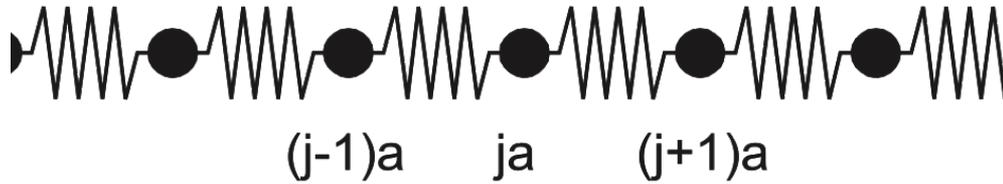
[http://www.research.ibm.com/DAMOCLES/html\\_files/soi.html](http://www.research.ibm.com/DAMOCLES/html_files/soi.html)

IBM Research Journal, DOI: 10.1147/rd.443.0323

# Phonons: atomic vibrations



# Phonon dispersion I

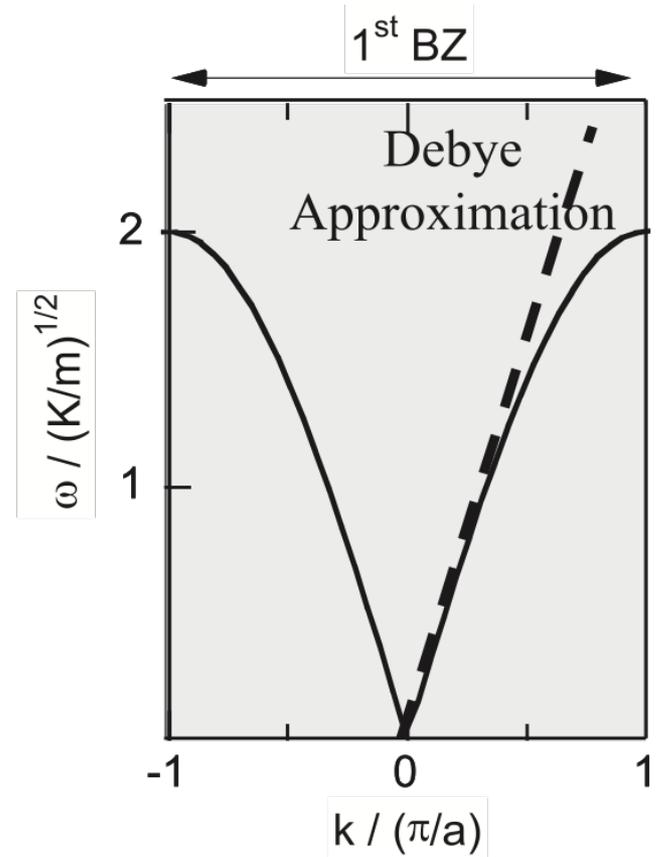


$$m \frac{d^2 u_j}{dt^2} = K(u_{j+1} - u_j) - K(u_j - u_{j-1})$$

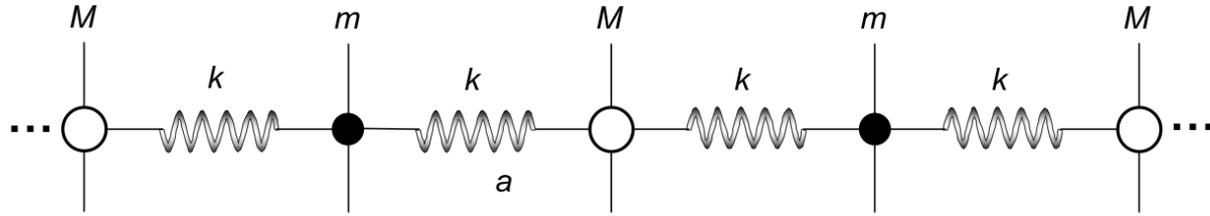
$$m \frac{\partial^2 u}{\partial t^2} = K a^2 \frac{\partial^2 u}{\partial x^2}$$

$$u_j = A \exp[-i(\omega t - kja)]$$

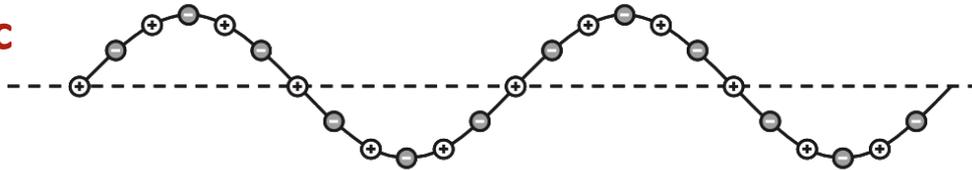
$$\omega = 2 \sqrt{\frac{K}{m}} \left| \sin \frac{ka}{2} \right|$$



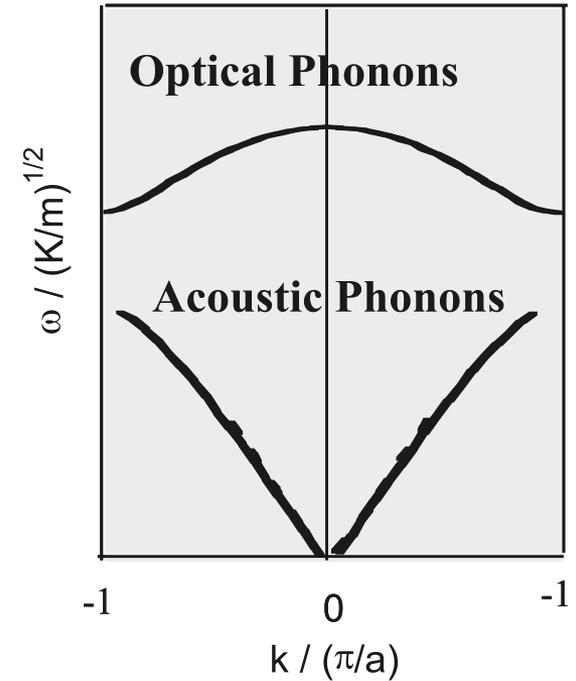
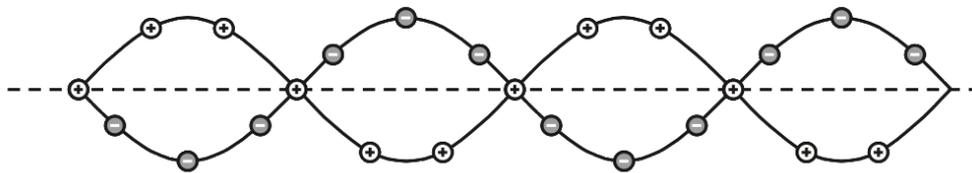
# Phonon dispersion II



**Acoustic**

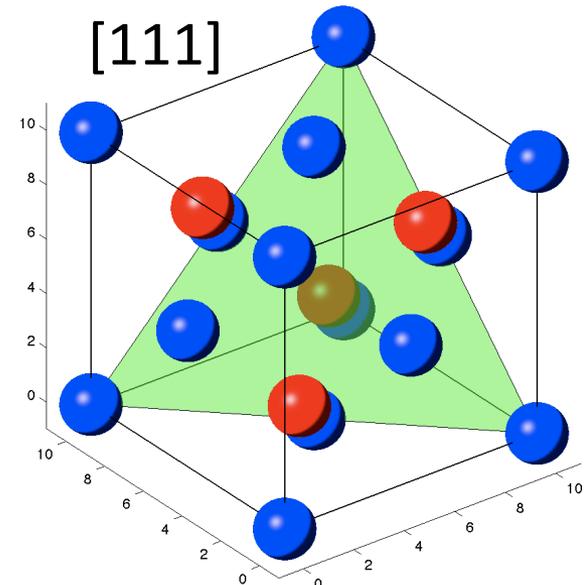
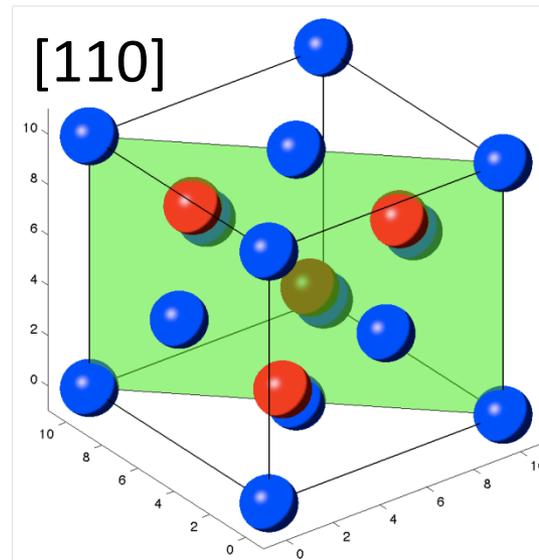
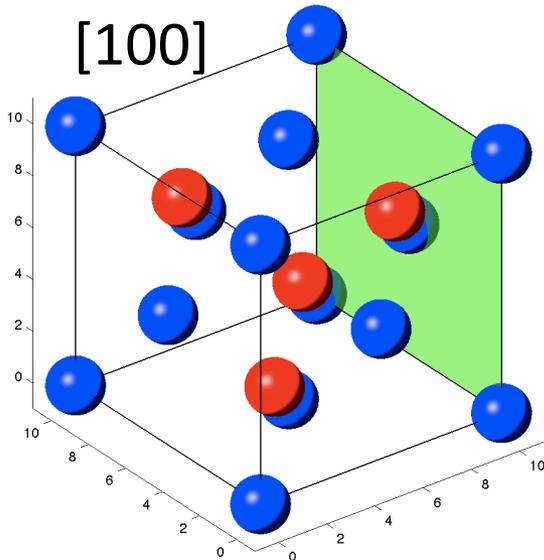
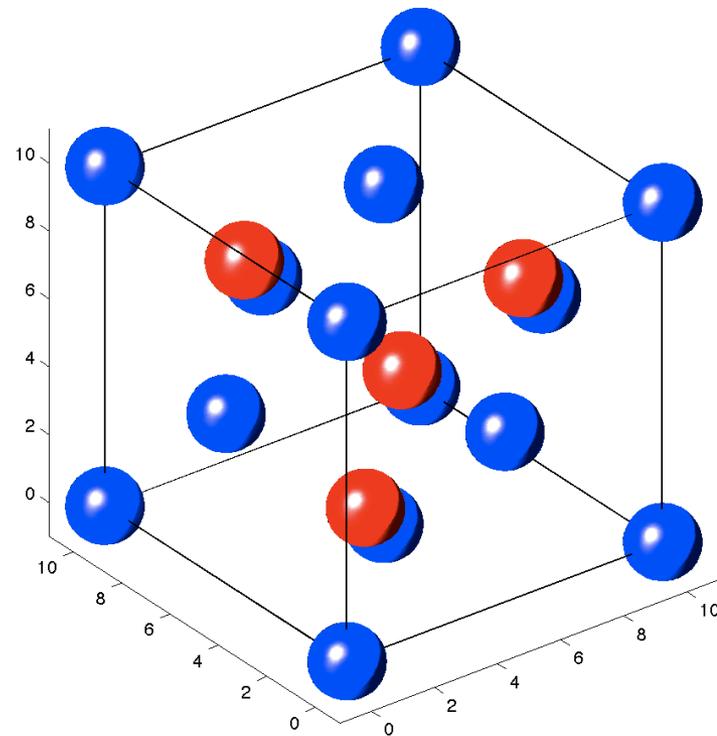


**Optical**

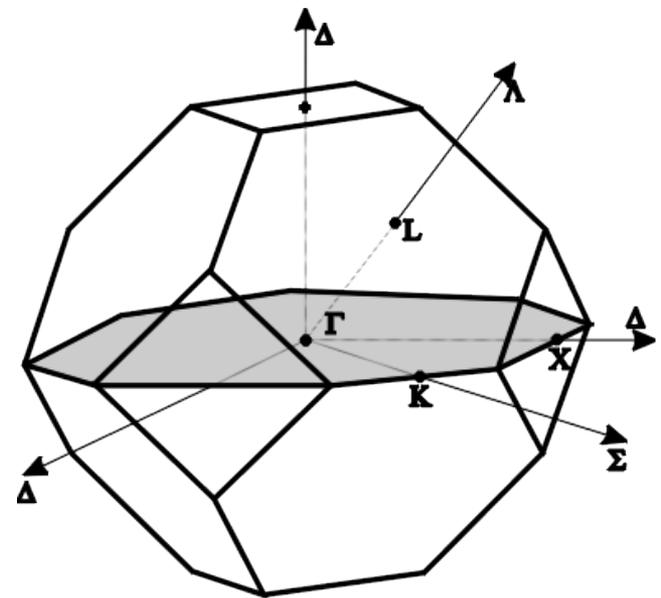
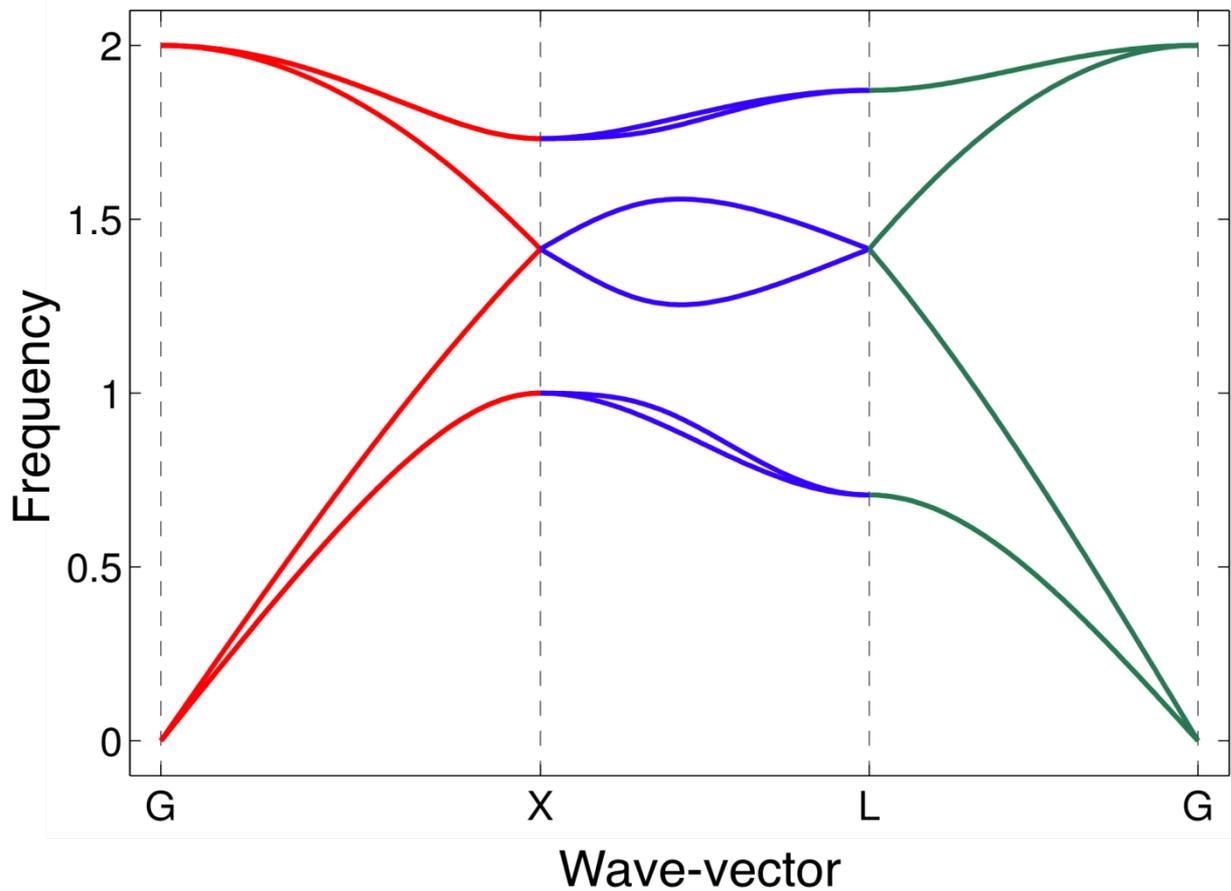
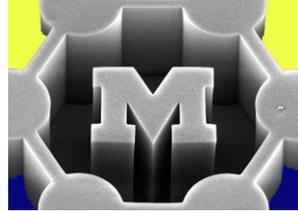


# A real crystal: InSb

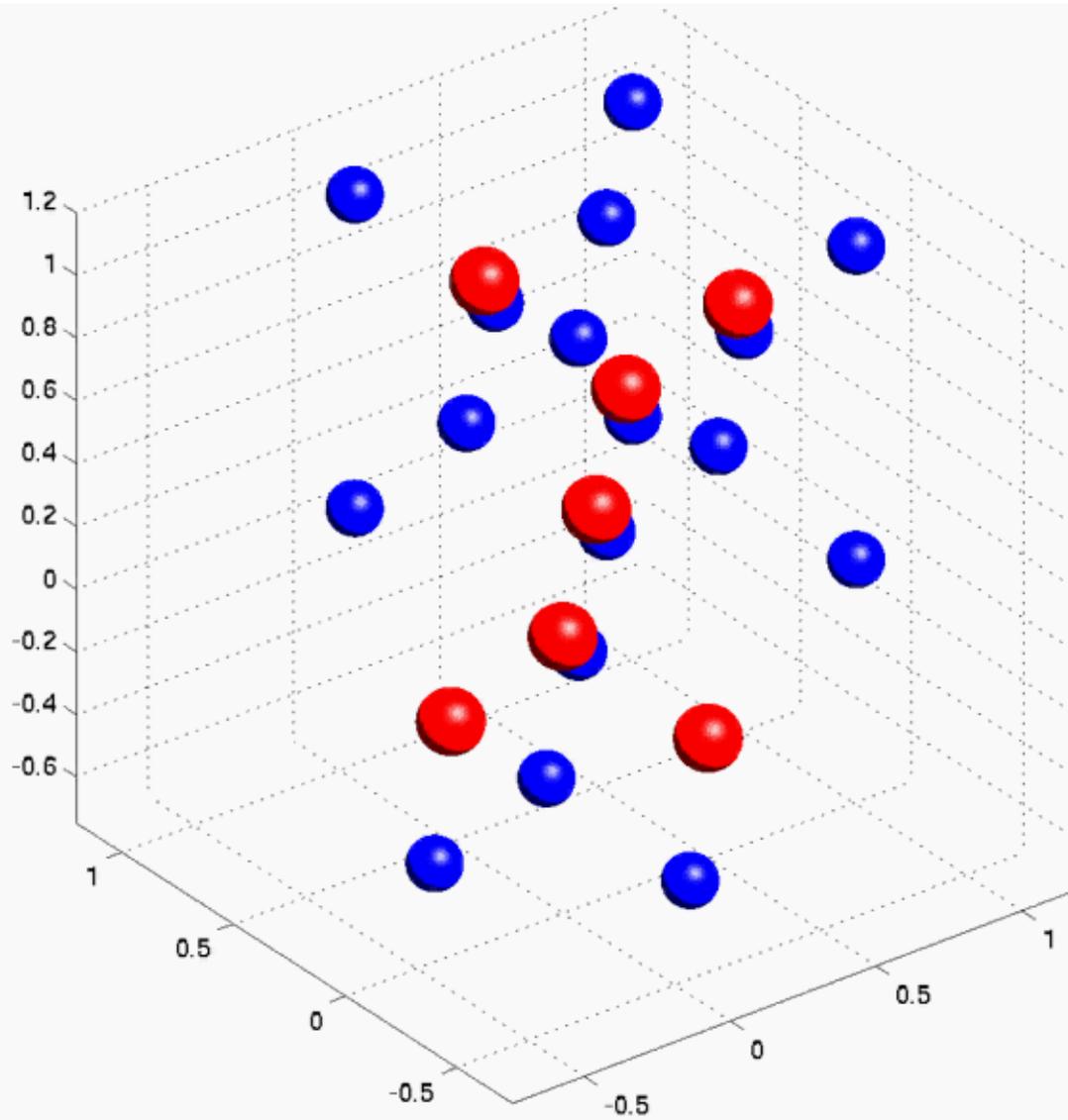
- "Zincblend" structure: FCC with diatomic basis
- Each atom feels stretch and bending force from neighbors



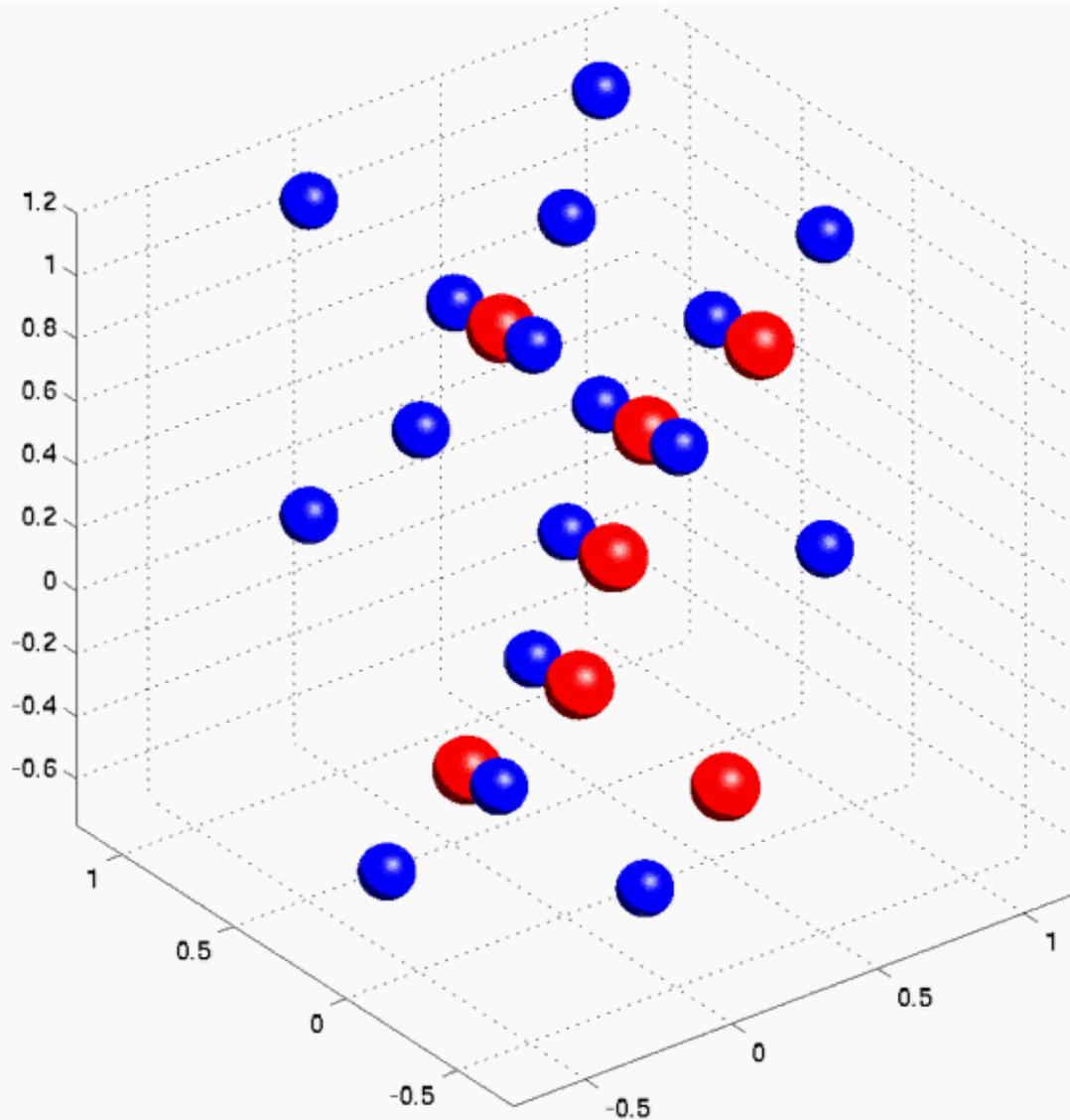
# Dispersion curves for InSb



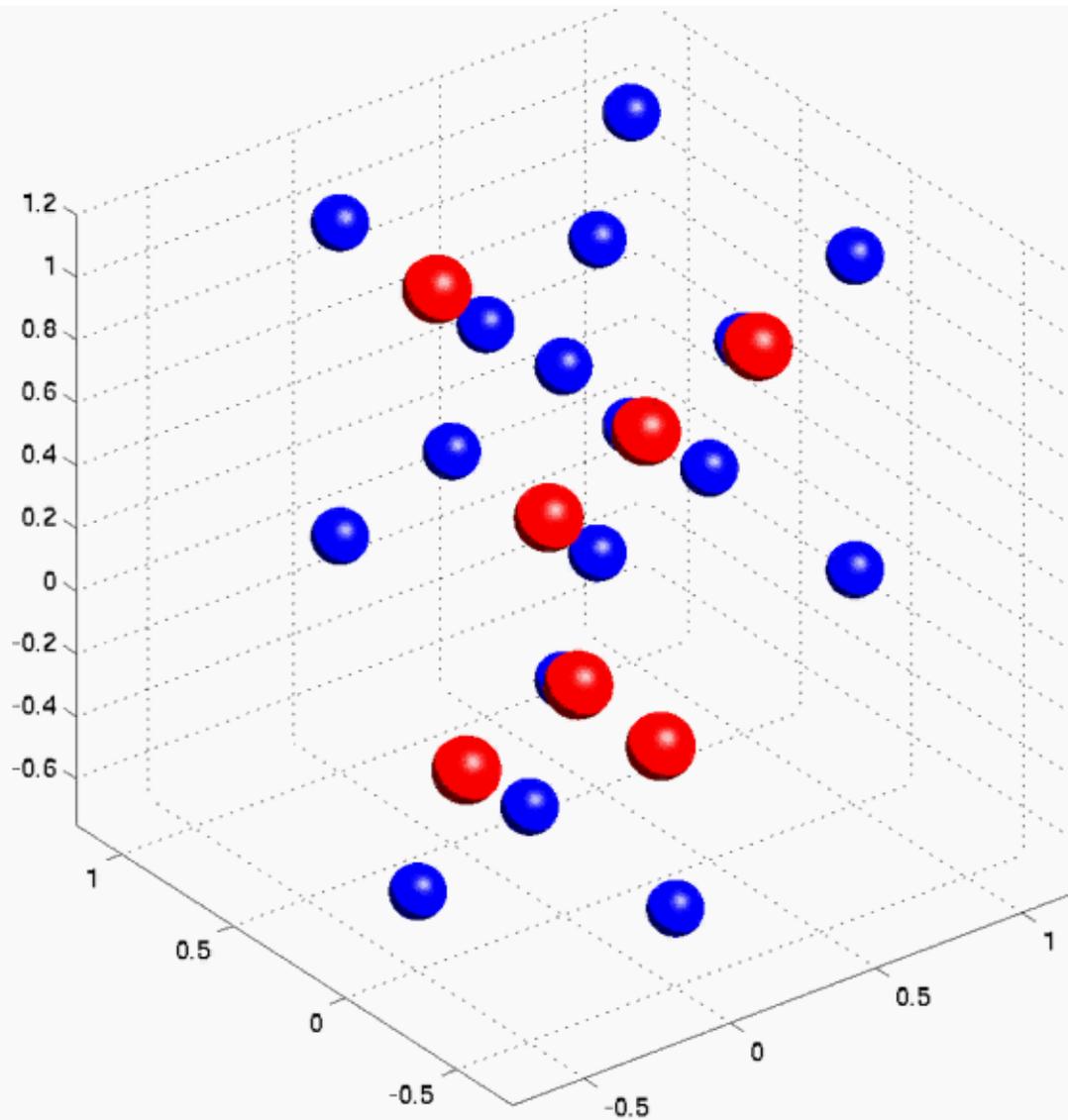
# [111] mode 1 (acoustic or optical?)



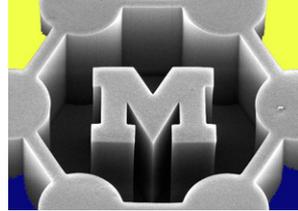
# G point, mode 4 (acoustic or optical?)



# X point, mode 6



# Phonons: quantized vibrations



- Energy (amplitude) of each mode comes in discrete units:

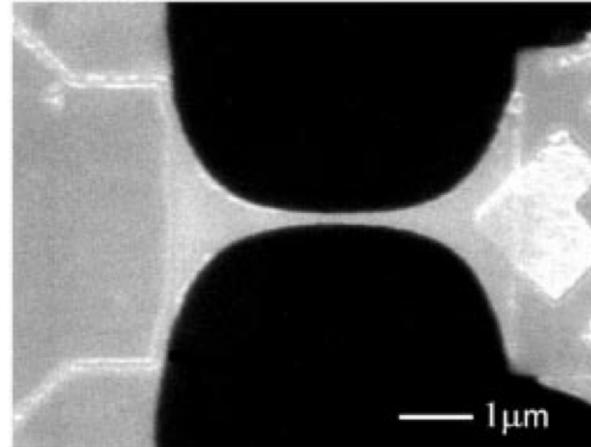
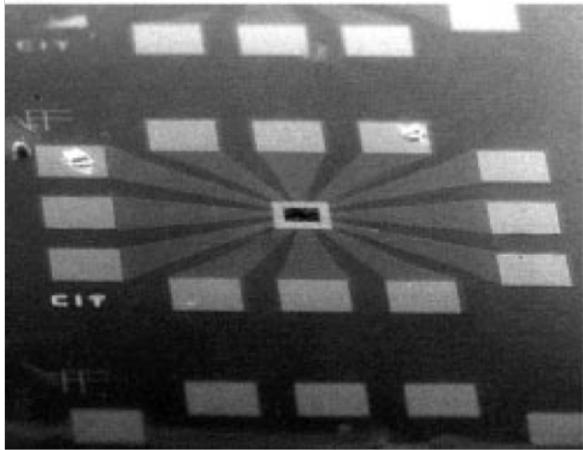
$$E_n = h\nu \left( n + \frac{1}{2} \right) \quad (n = 0, 1, 2, 3, \dots)$$

- The modes obey Bose-Einstein statistics (like photons)
- Quantum of thermal conductance:

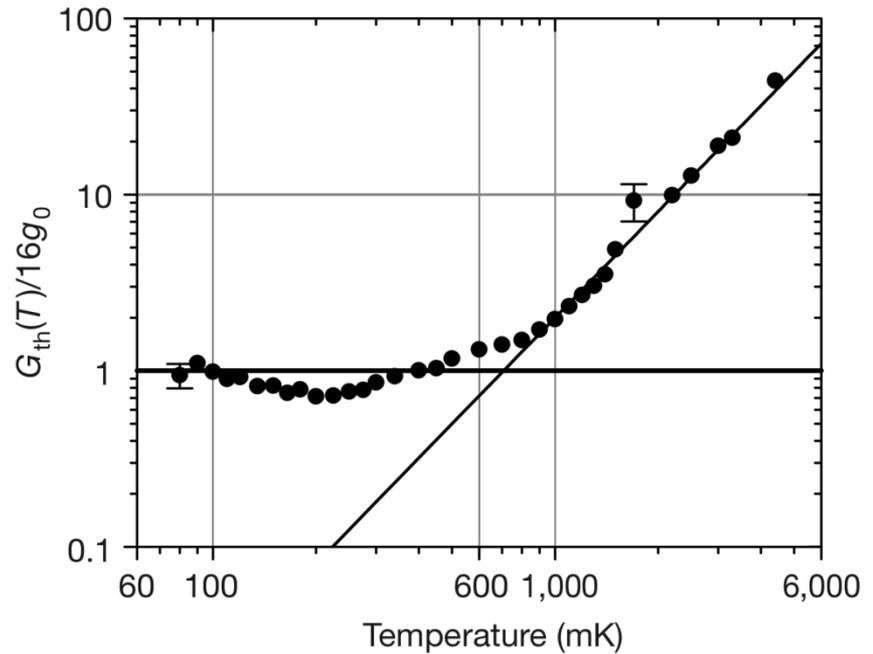
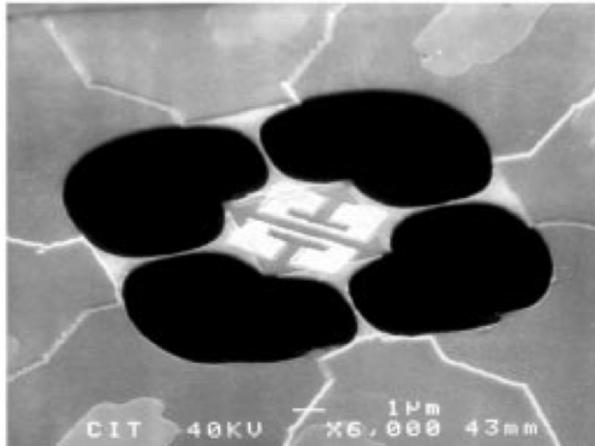
$$G_{\text{th}} = g_0 = \pi^2 k_B^2 T / (3h)$$

$$g_0 = (9.456 \times 10^{-13} \text{ W/K}^2) T$$

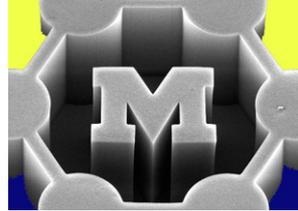
# Measuring the quantum of thermal conductance



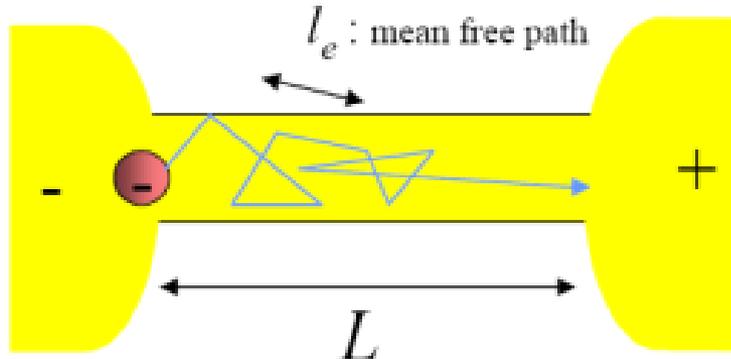
Mean free path  
 $\approx 1 \mu\text{m}$



# Diffusive vs. ballistic transport



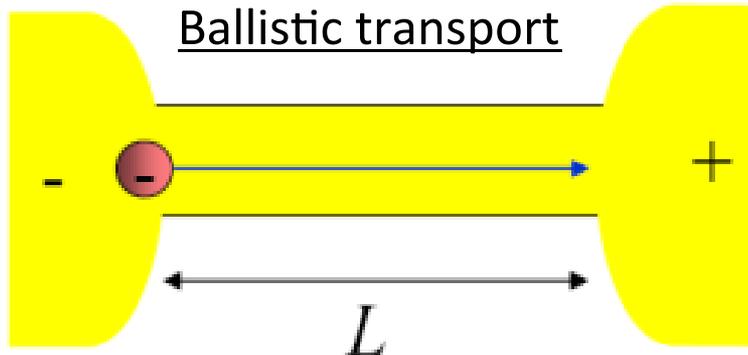
## Diffusive transport



$$l_e \ll L \quad R(L) = rL$$



## Ballistic transport



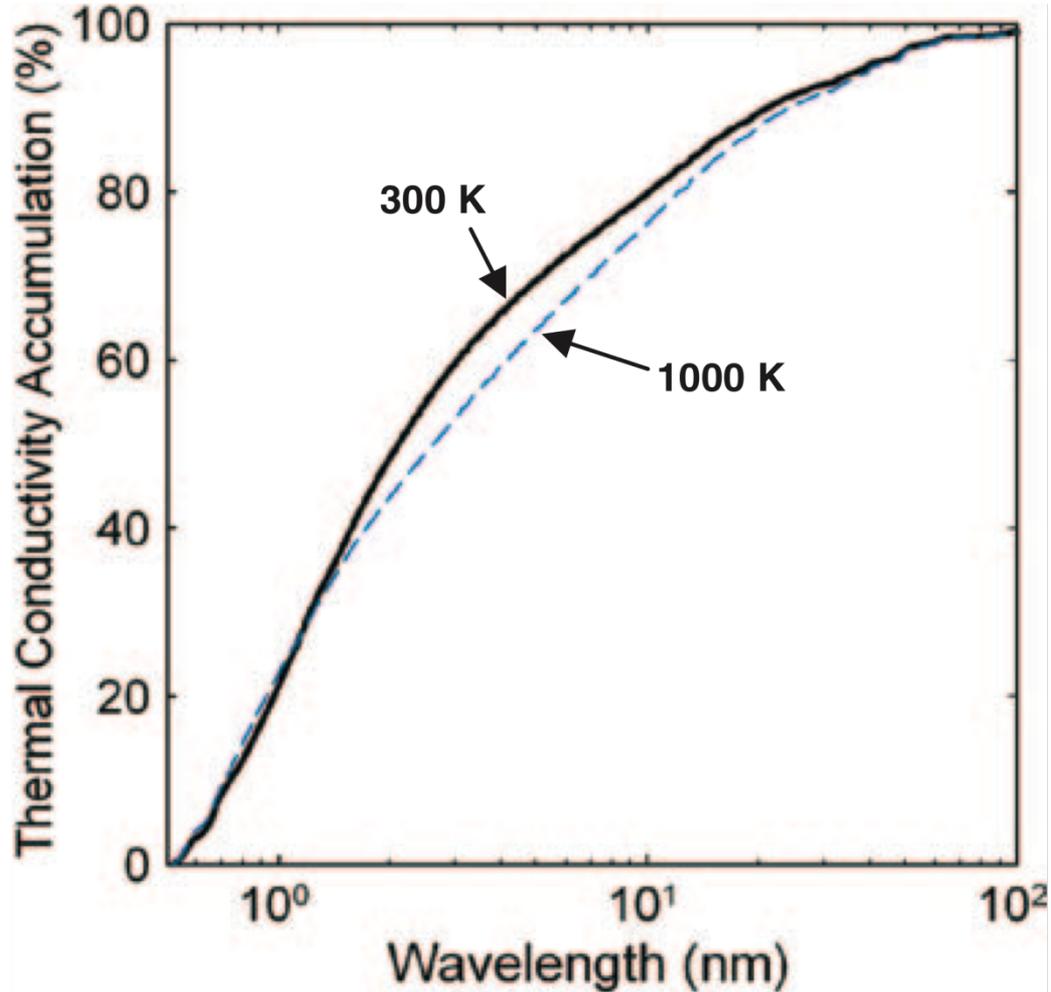
$$L < l_e \quad R(L) = R_Q$$



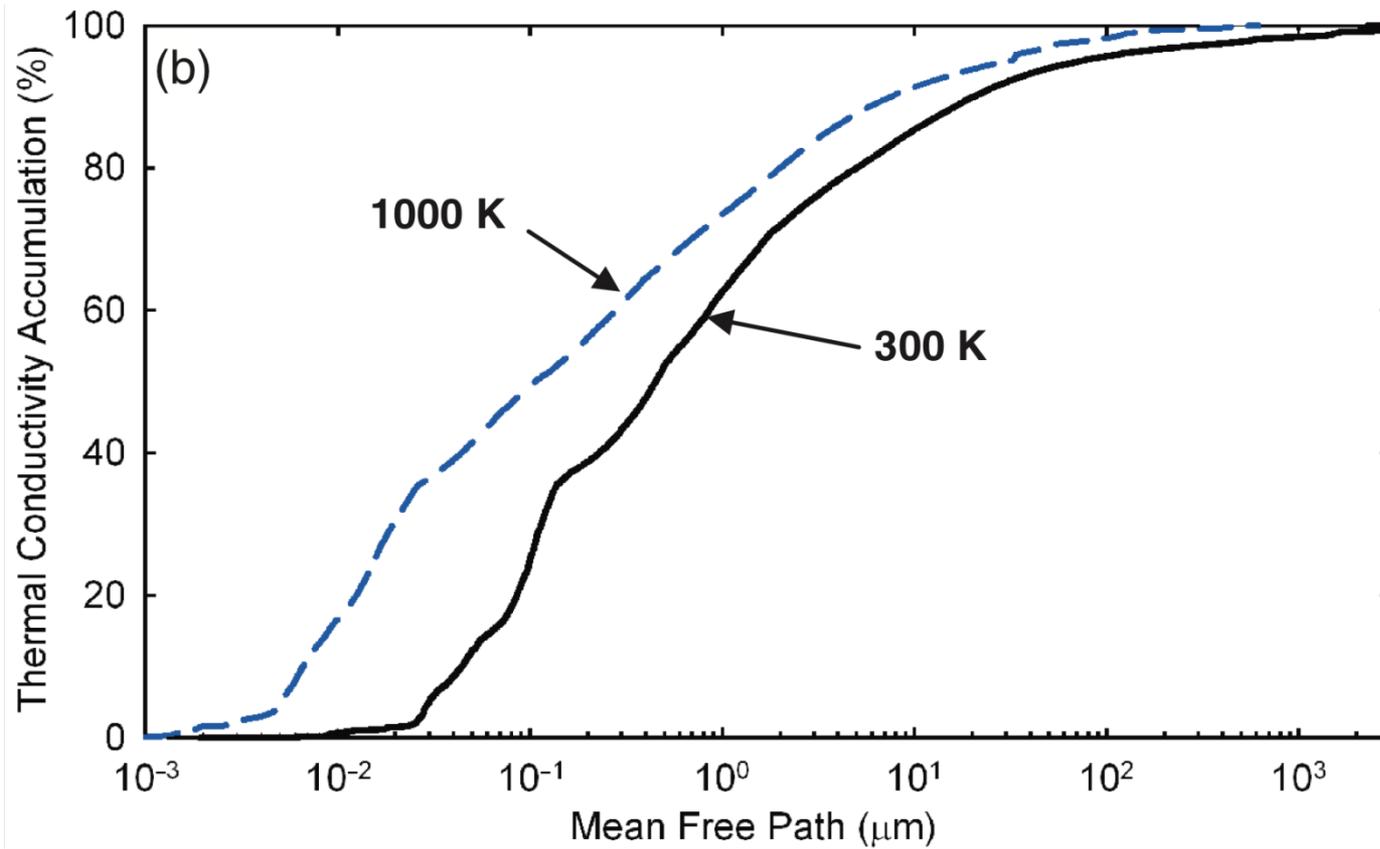
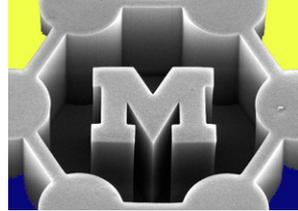
(P. Kim, @NT'06)

(J. Chen, IBM)

# Silicon: thermal conductivity vs. phonon wavelength

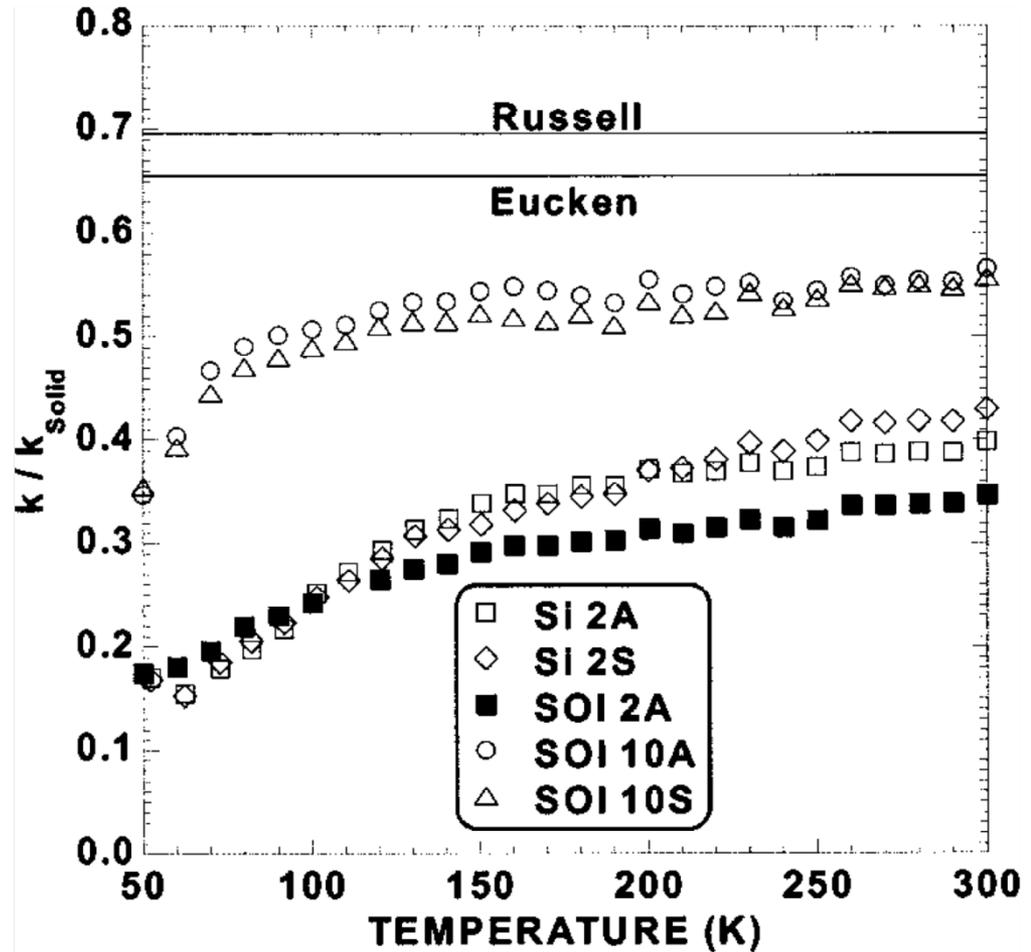
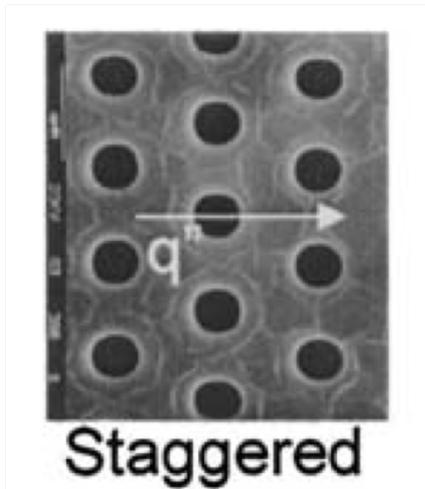
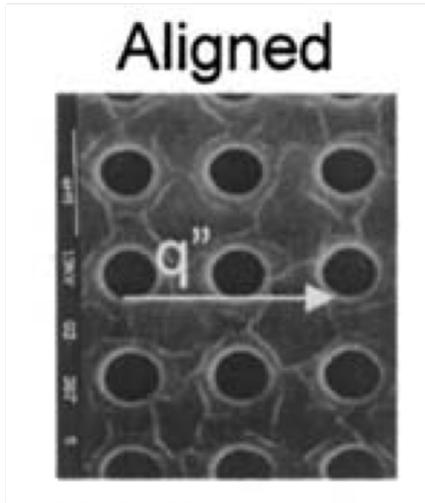
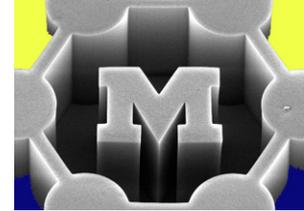


# Silicon: thermal conductivity vs. MFP



At room temperature, phonons with MFP  $> 1 \mu\text{m}$  contribute 30% to thermal conductivity!

# Effect of micro-holes in Si membranes



# Thin films: phonon scattering at boundaries reduces thermal conductivity

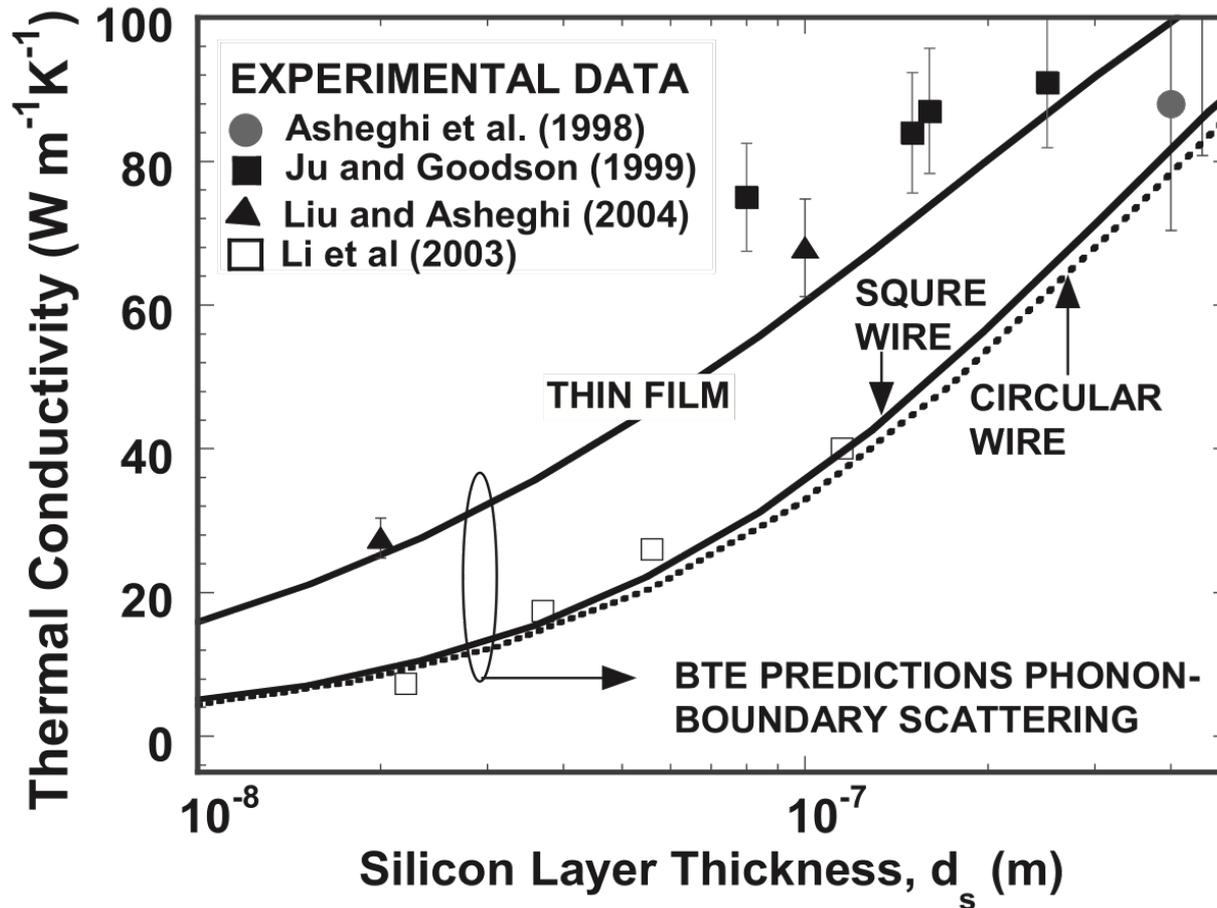
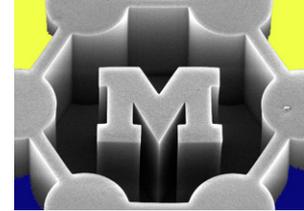


Figure 1.15 Thermal conductivity of silicon films as a function of the film thickness or wire diameter. (Courtesy of M. Ashegli).

# Individual suspended tubes and wires

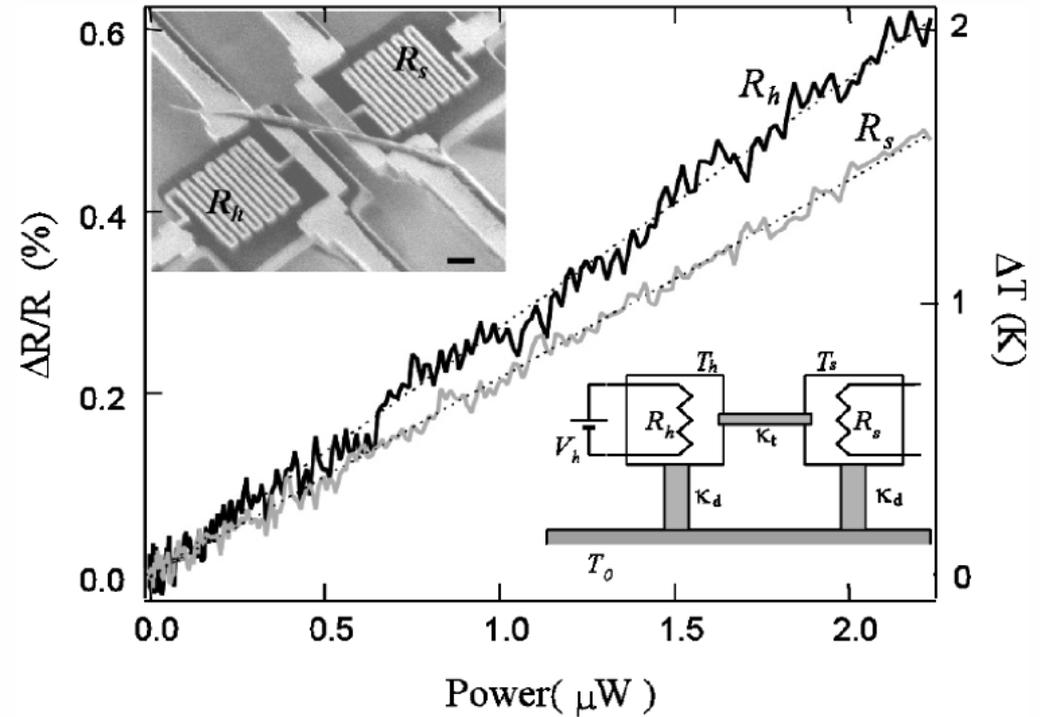
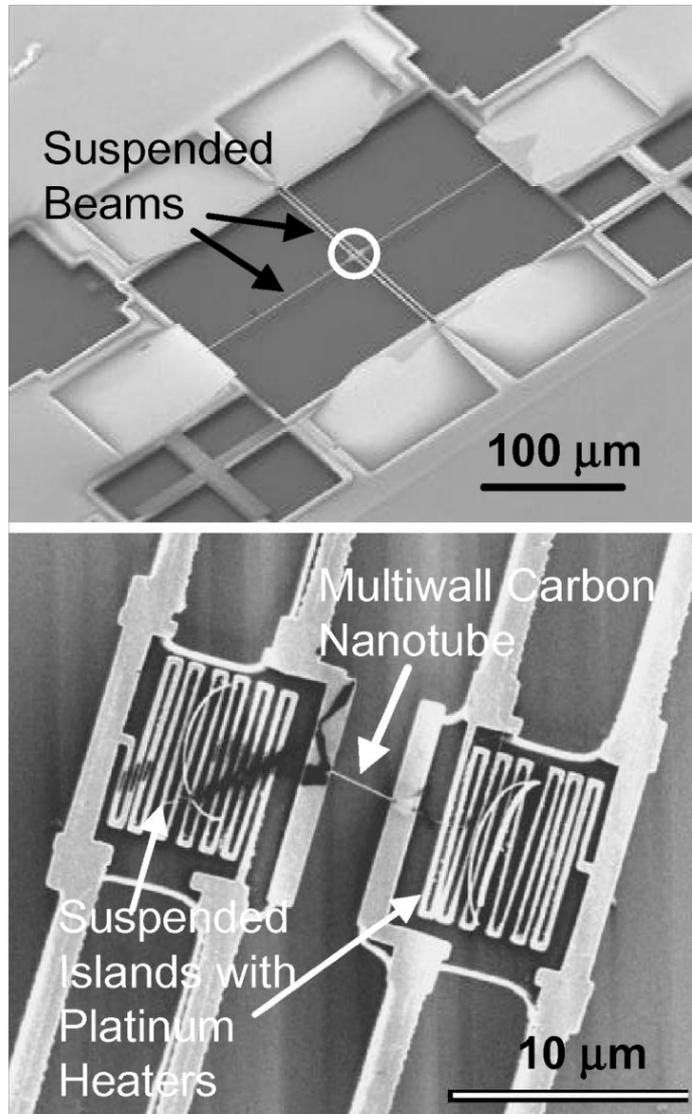
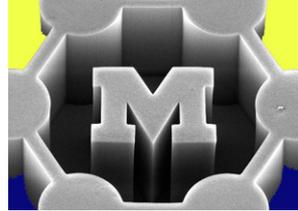


FIG. 2. The change of resistance of the heater resistor ( $R_h$ ) and sensor resistor ( $R_s$ ) as a function of the applied power to the heater resistor. Upper inset: SEM image of the suspended islands with a MWNT bundle across the device. The scale bar represents 1  $\mu\text{m}$ . Lower inset: A schematic heat flow model of the device.

# Individual suspended MWNTs

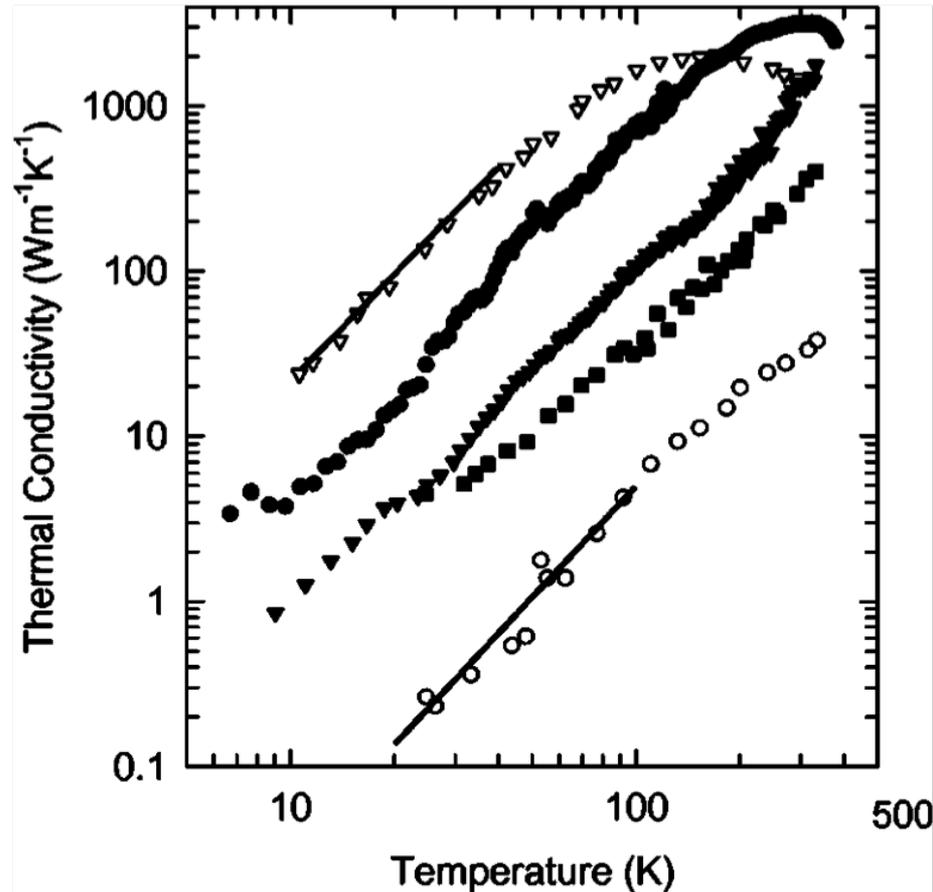
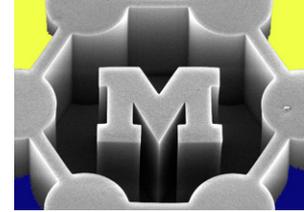
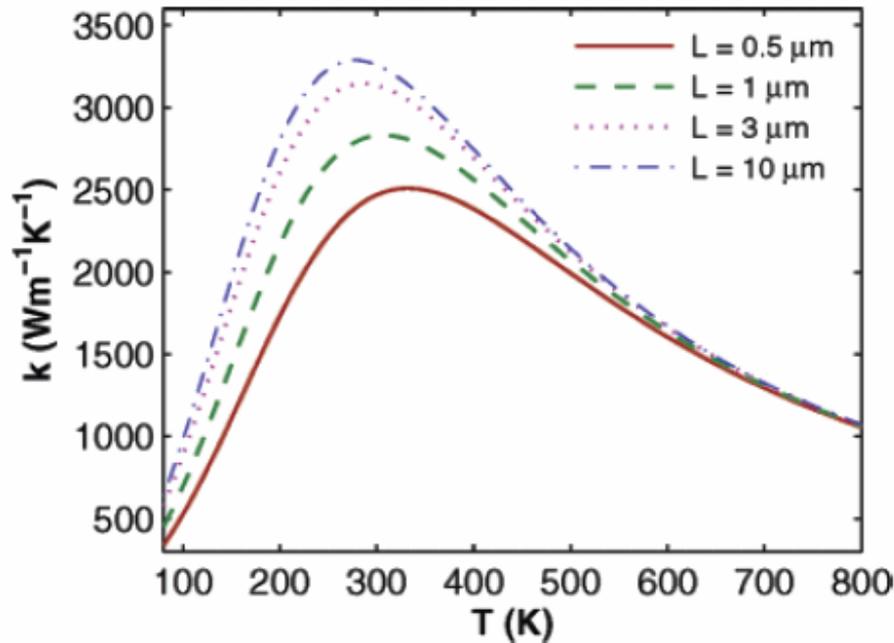
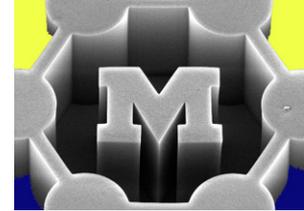


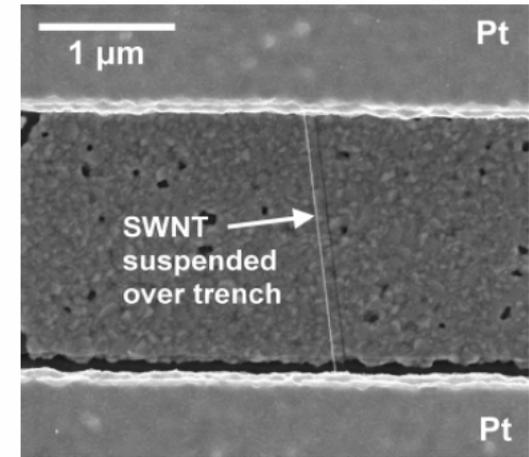
FIG. 9. Measured thermal conductivity of a 14 nm diameter multiwall carbon nanotube (MWCN) (solid circle), an 80 nm diameter MWCN bundle (solid triangle), and a 200 nm diameter MWCN bundle (solid square) (Ref. 94). Data for two vapor-grown graphite fibers (Ref. 95), one heat treated to 3000 °C (open triangle) and one without heat treatment (open circle) are included for comparison. The lines represent the calculated (Ref. 95) basal-plane thermal conductivity of graphite, assuming temperature-independent low-temperature phonon mean free path  $\ell = 2.9 \mu\text{m}$  (upper line) and  $\ell = 3.9 \text{ nm}$  (lower line).

# Individual suspended SWNT



**Figure 6.** Analytic plot of the intrinsic SWNT thermal conductivity over the 100–800 K temperature range as computed with eq 3. The length dependence is included heuristically with a simple scaling argument, but differences in chirality may lead to variations up to 20% between different tubes.<sup>11</sup>

$$k(L, T) = [3.7 \times 10^{-7}T + 9.7 \times 10^{-10}T^2 + 9.3(1 + 0.5/L)T^{-2}]^{-1} \quad (3)$$

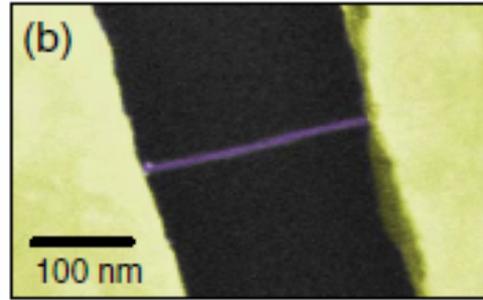
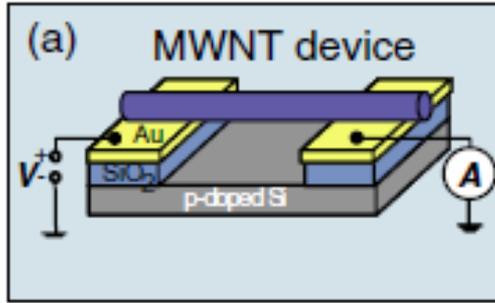


**Figure 1.** Scanning electron microscope (SEM) image of a typical SWNT freely suspended across a 2- $\mu\text{m}$  trench and lying on top of the Pt contacts. This sample was coated with 1.5 nm Ti/2.5 nm Au to facilitate SEM imaging.

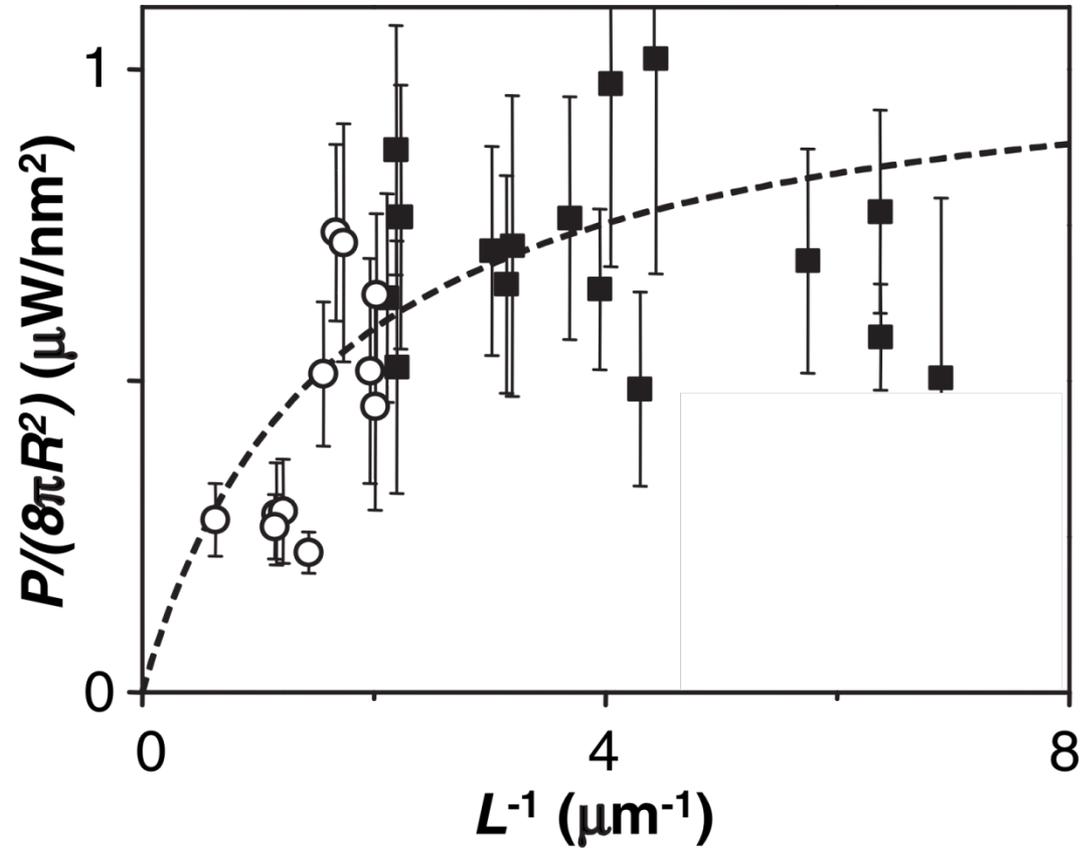
Phonon mean-free path  
at 300 K



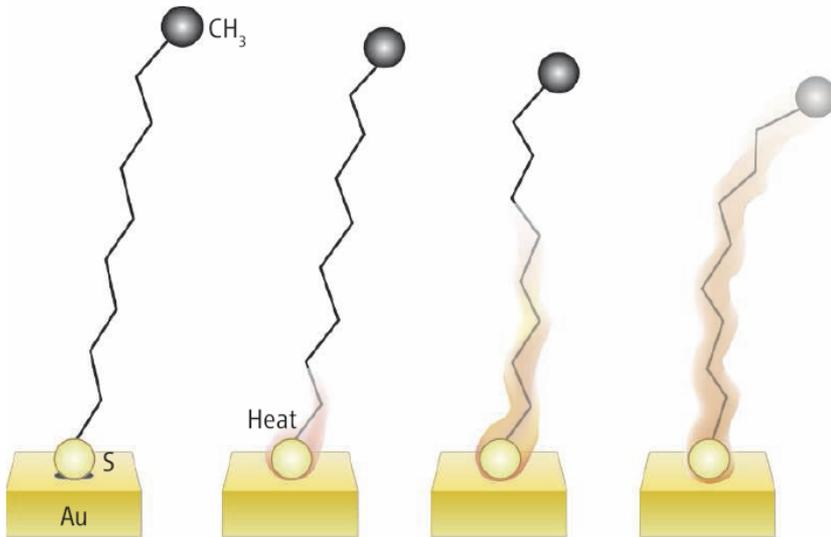
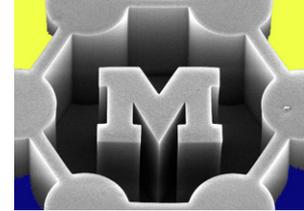
# Ballistic phonon transport in MWNTs



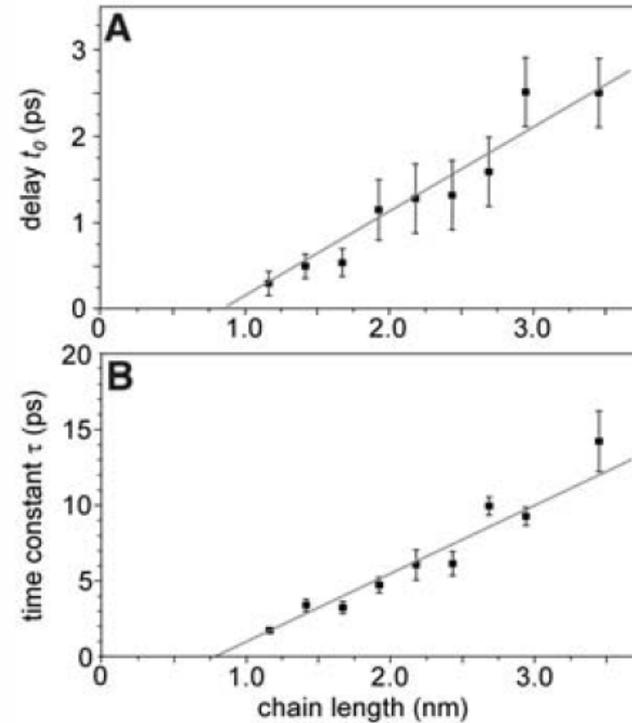
Power independent  
of length below  
500nm



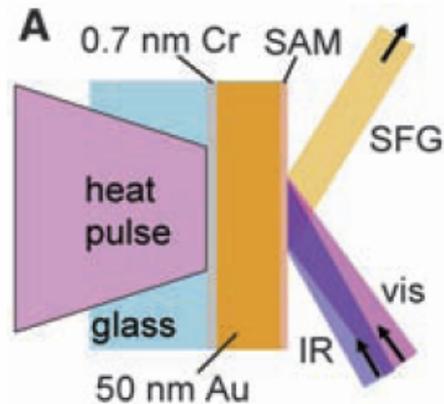
# Heat conduction in molecular chains



**How molecules heat up.** In the experiments reported by Dlott and co-workers, heat is transferred from the heated gold substrate along the molecular chain, causing the chain to become increasingly disordered.



**Fig. 4.** (A) Dependence on chain length of the delay time  $t_0$  between the flash-heating pulse and the arrival of the initial burst of heat at the methyl head groups. (B) Dependence on chain length of the time constant  $\tau$  for thermal equilibration between flash-heated Au and alkane chains.

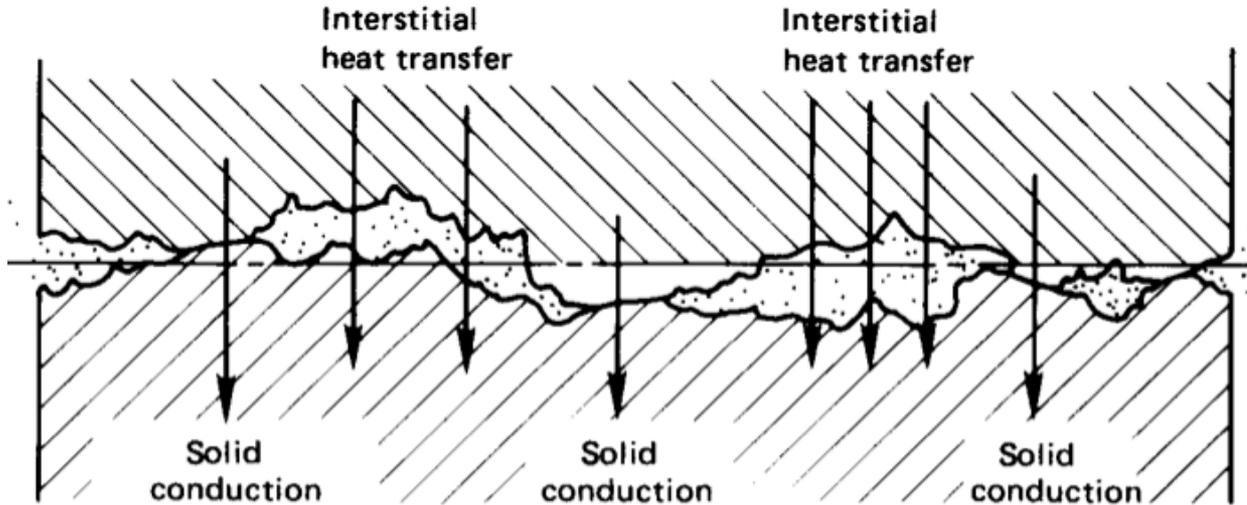


→ Ballistic transport, velocity  $\approx 1$  m/s

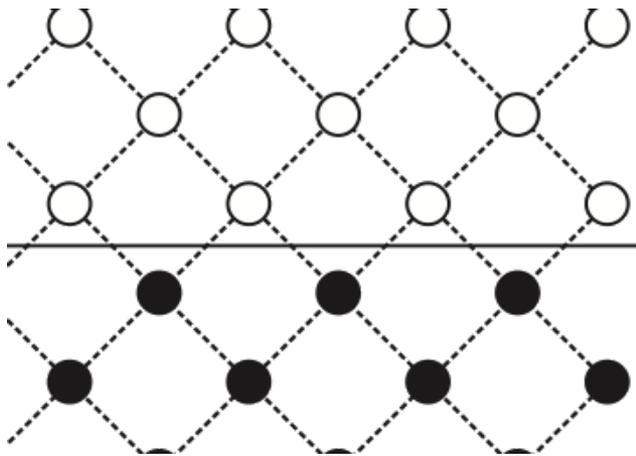
# Thermal boundary conductance



$$q = G\Delta T$$



Caused by  
gaps



Caused by reflection and  
transmission of phonons at  
interfaces

# Simple models for G



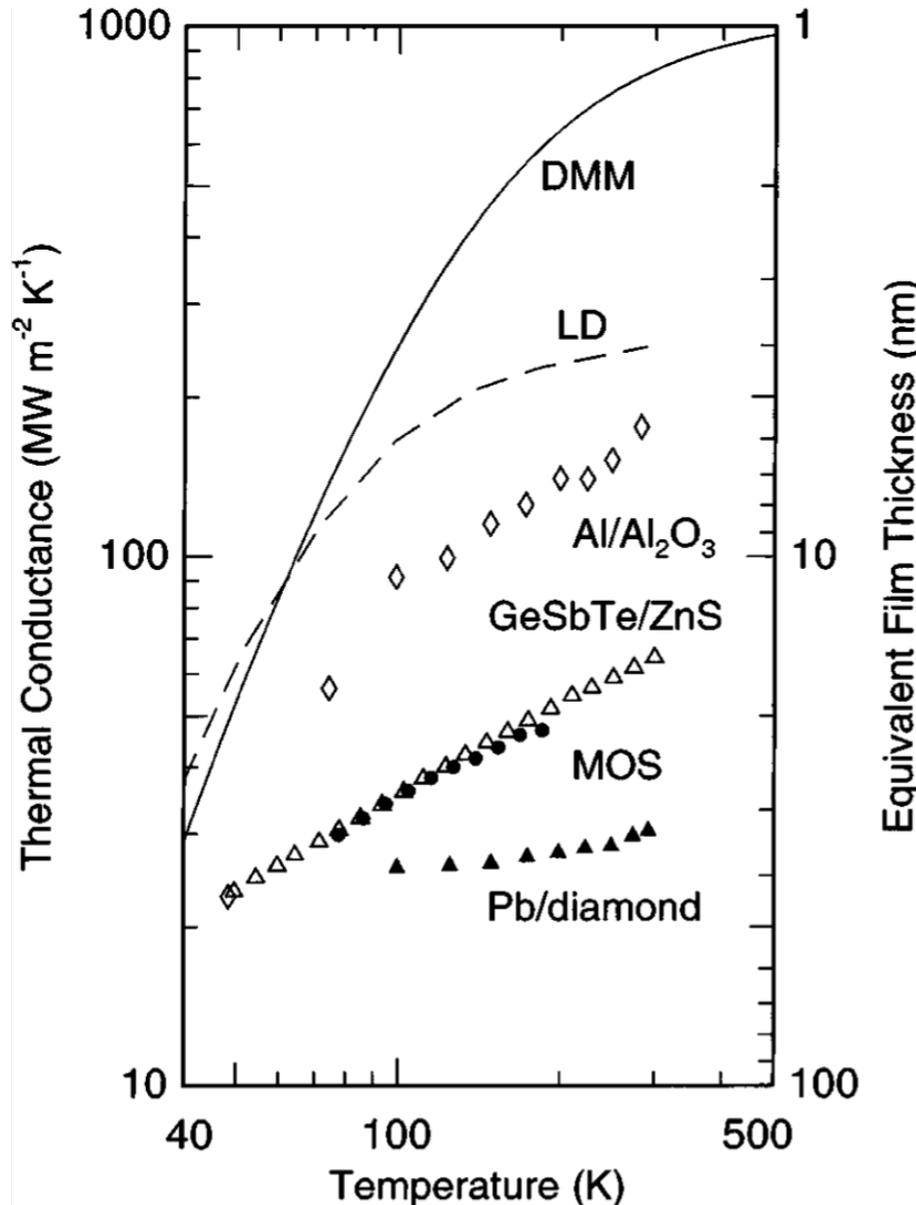
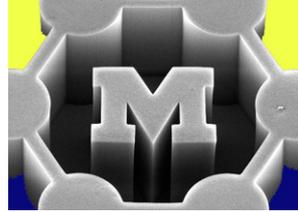
- Caused by reflection and transmission of phonons at interfaces
- Mismatch of phonons
  - Acoustic mismatch (AMM): all phonons scattered specularly at an interface (Snell's law)
  - Diffuse mismatch (DMM): all phonons scattered diffusely (no "memory" of where they came from)
- Diffuse mismatch is better at room temperature (even 1 atom roughness can scatter thermal phonons)

$$G_e = \frac{1}{4} \sum_j^3 \int_0^{\omega_{\max,j}} \alpha_{1-2} \hbar \omega v_1 D_1 \frac{\partial f}{\partial T_e} d\omega$$

The equation is annotated with red boxes and lines pointing to labels:

- $\alpha_{1-2}$ : transmissivity
- $\hbar \omega$ : phonon energy
- $v_1$ : phonon speed
- $D_1$ : DOS (Density of States)
- $\frac{\partial f}{\partial T_e}$ : occupation function

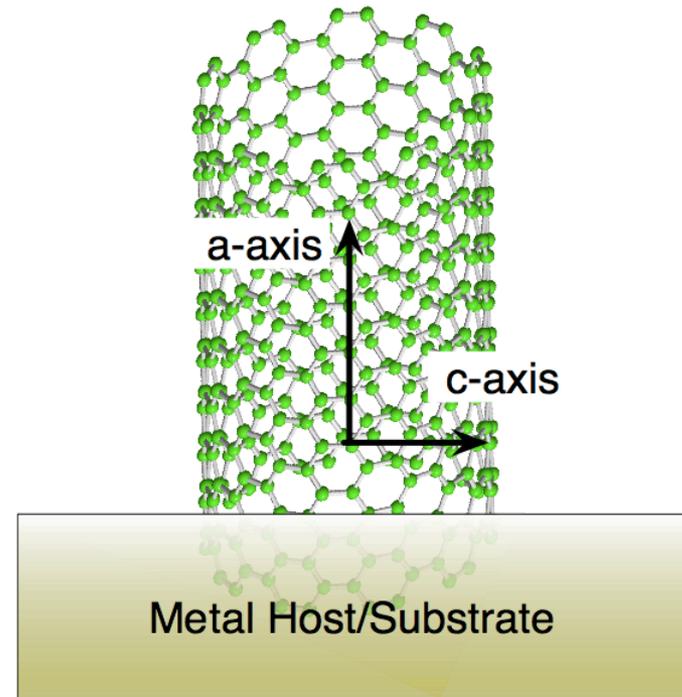
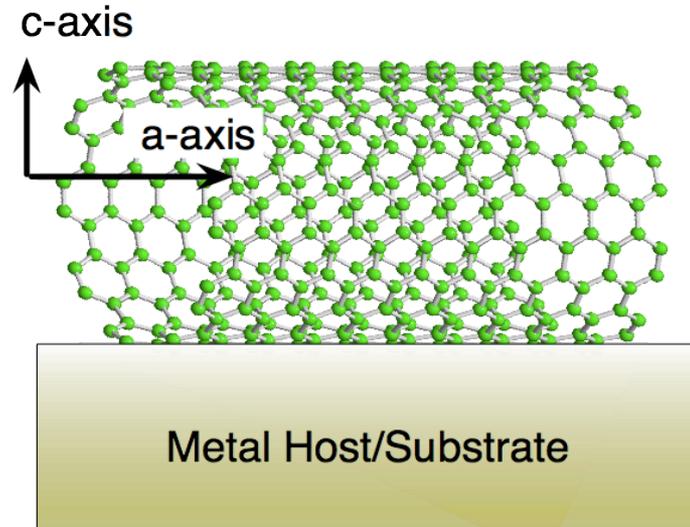
# Some values of G



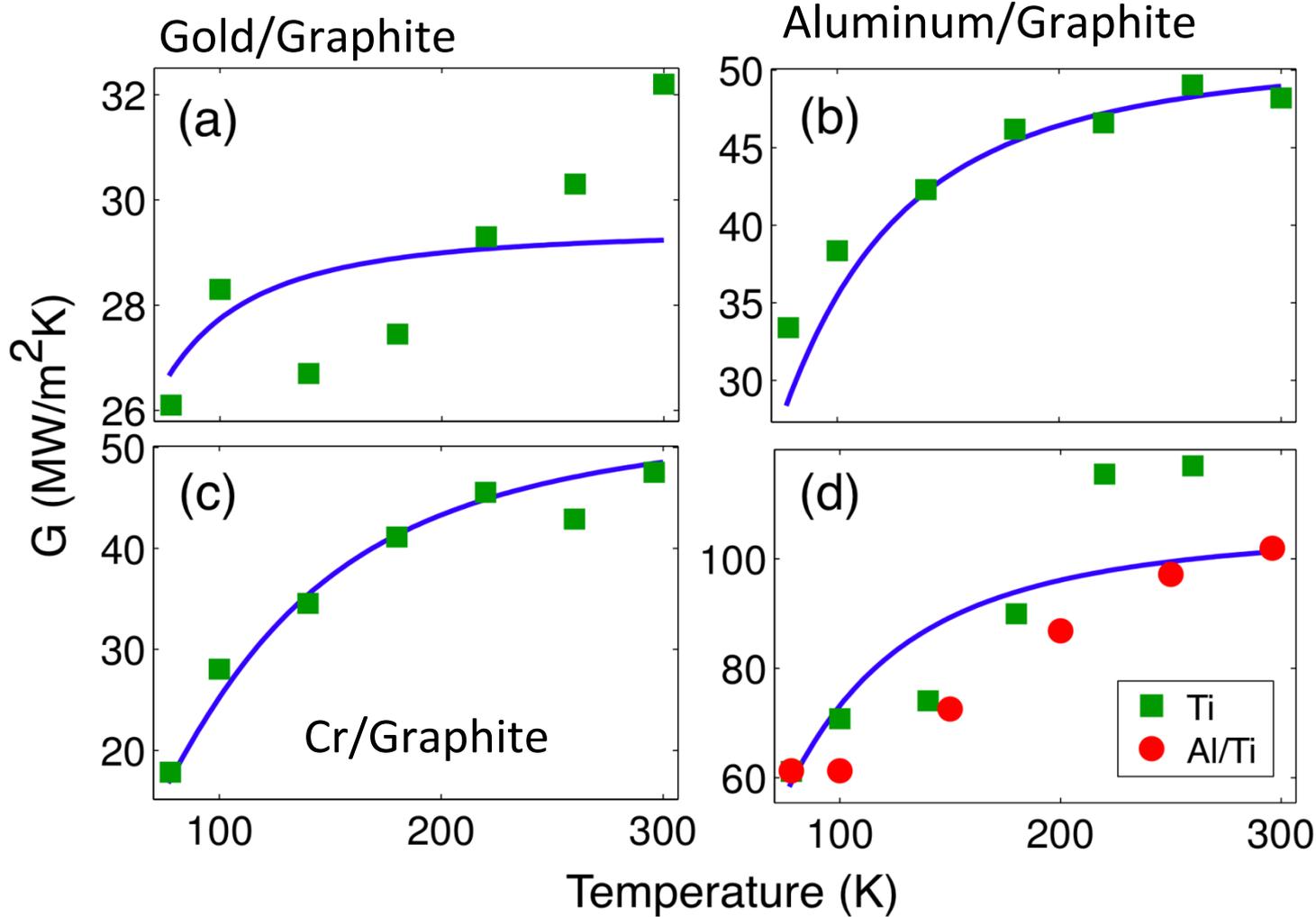
# An example: metal-graphite interfaces



- Graphite-based electronics
- Thermal interface materials
- Characterization of carbon nanostructures

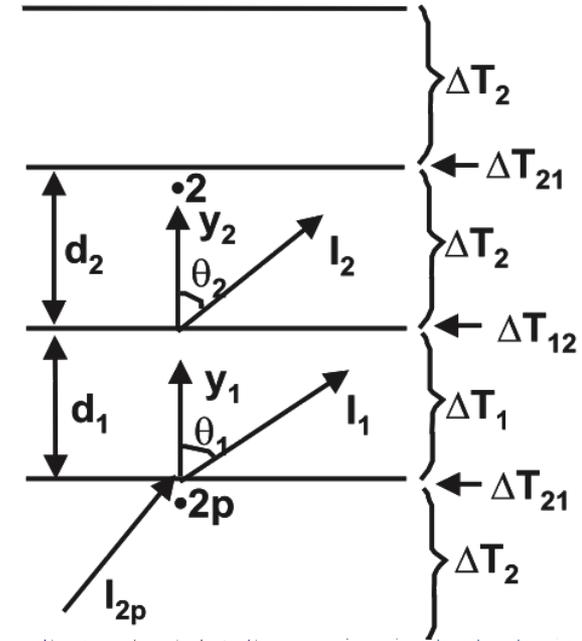
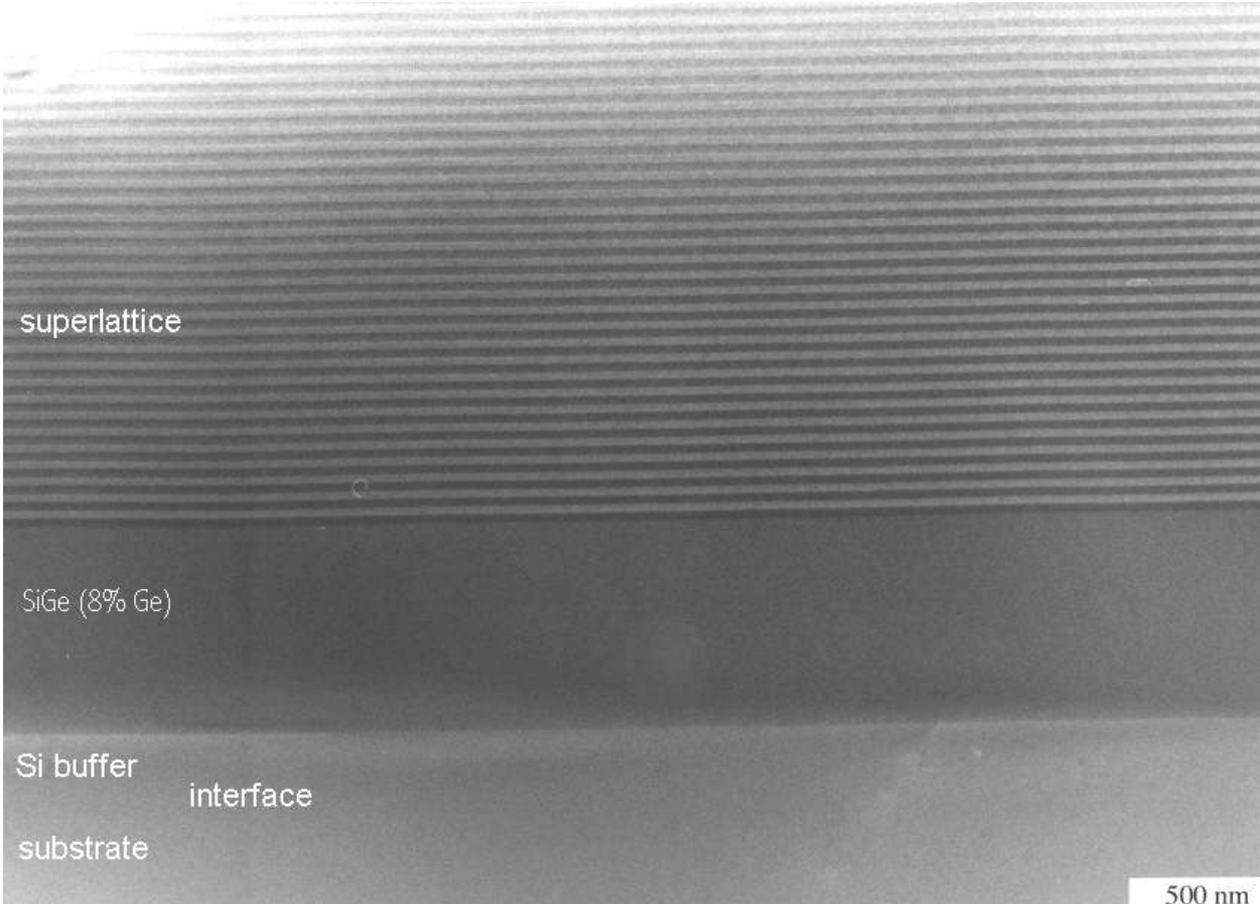


# Measurements and DMM predictions



Ti/Graphite

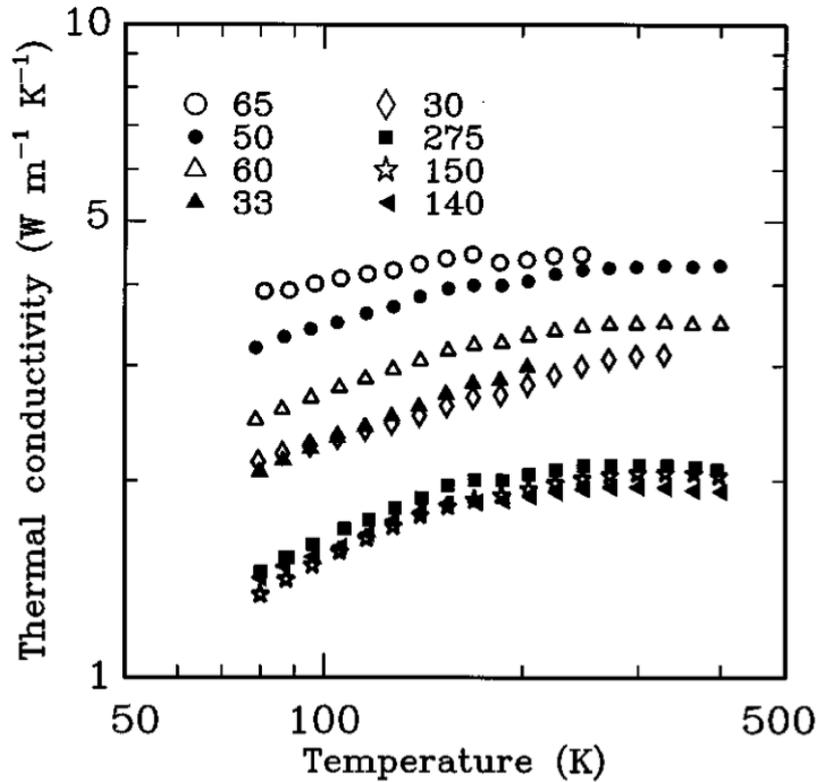
# Superlattices



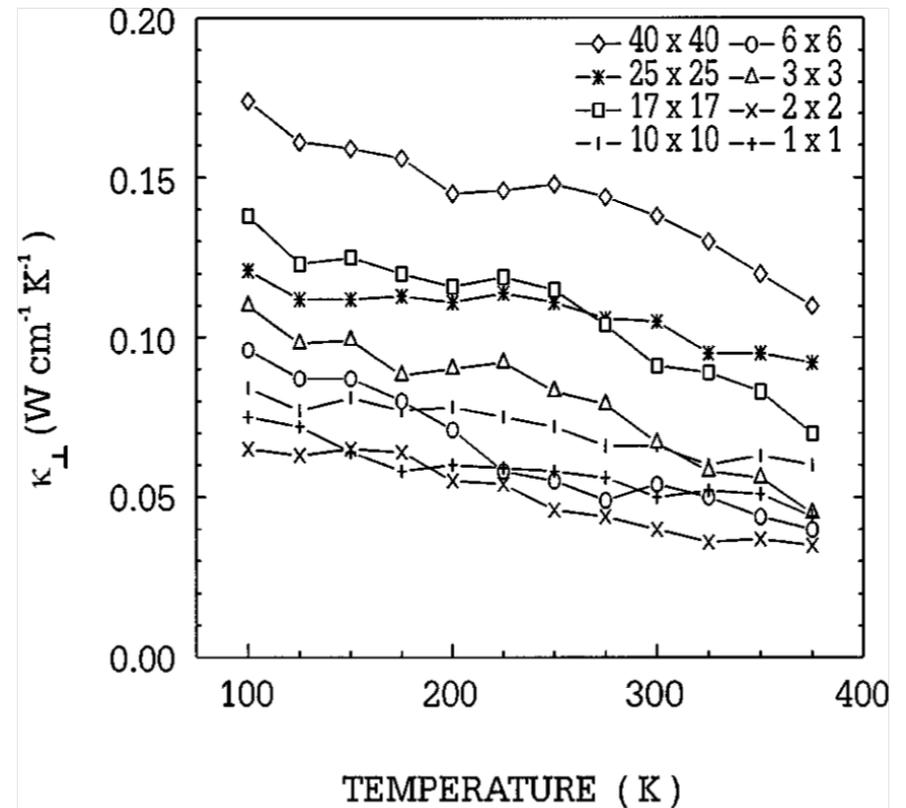
# Superlattices



## Si/Ge



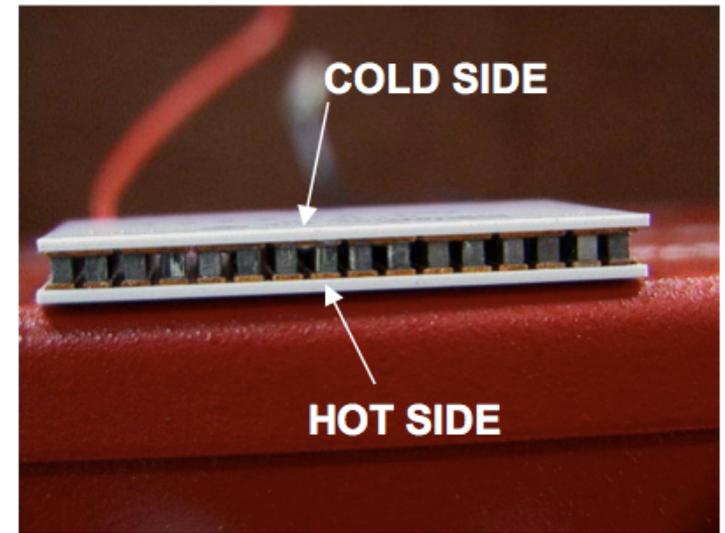
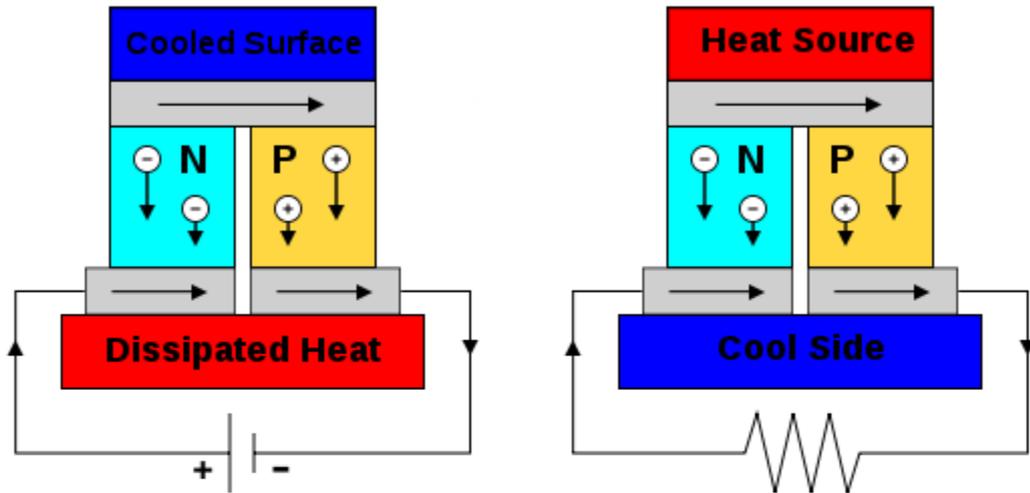
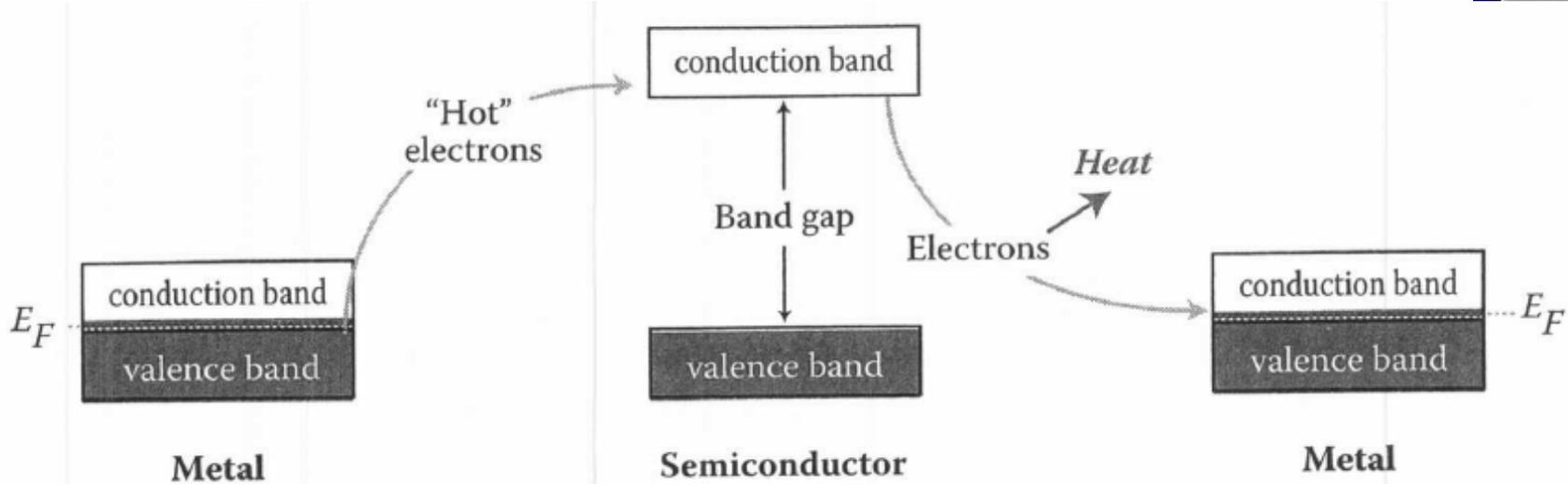
## GaAs/AlAs



# Thermoelectricity - heaters and coolers



- Voltage-induced separation of charge carriers → heat



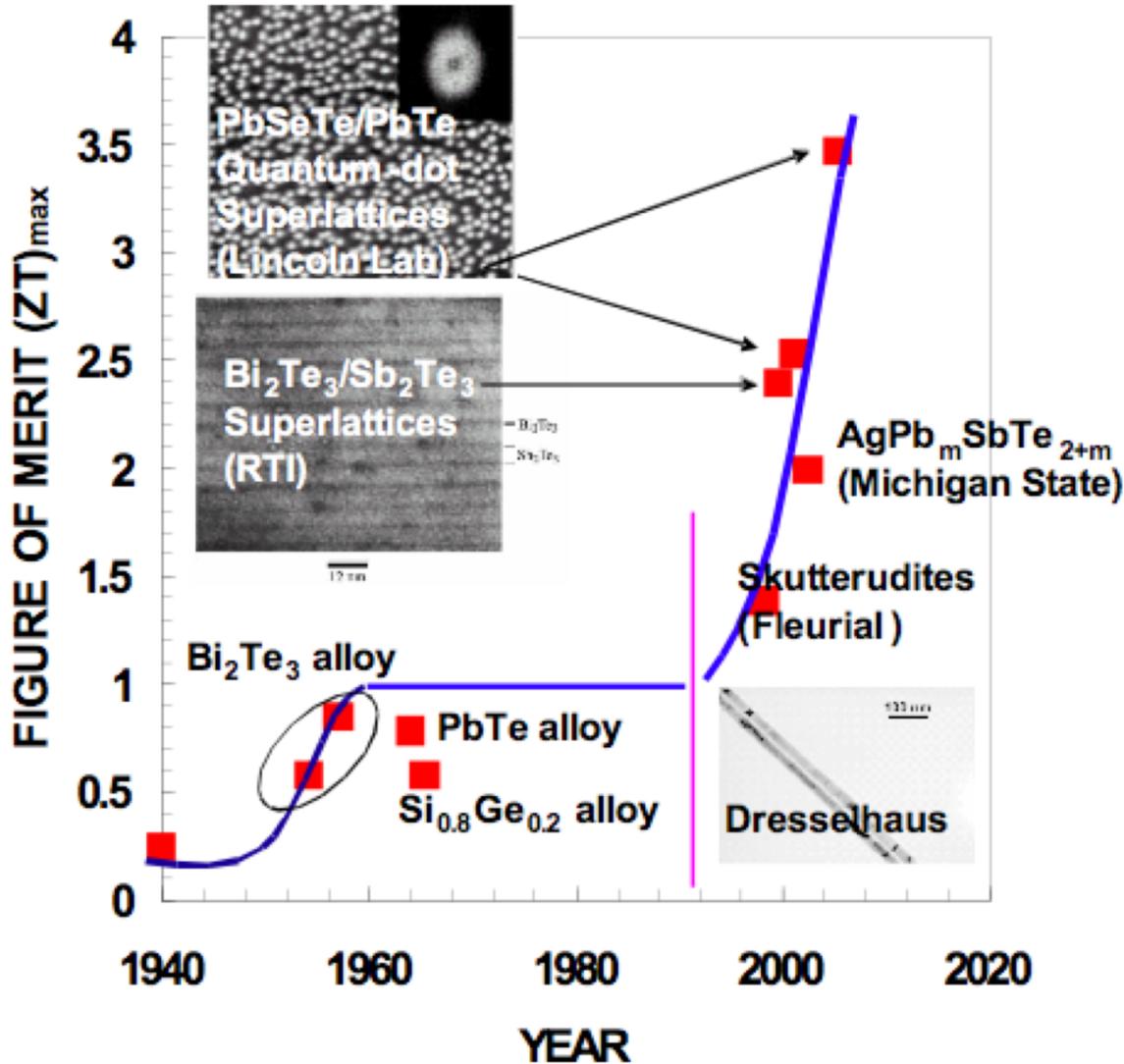
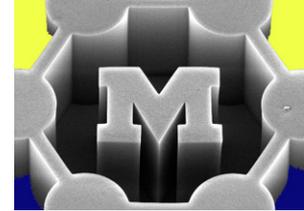
# Thermoelectric figure of merit



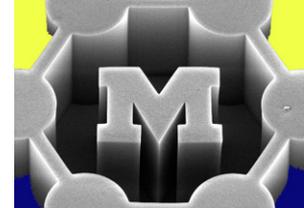
$$Z = \frac{S^2 \sigma}{k} \quad ZT = \frac{S^2 \sigma}{k} T$$

- Need  $ZT \approx 2-3$  to compete with conventional technology (e.g., refrigerators)
- Seebeck coefficient ( $S$ ) measures voltage response to applied temperature difference
  - $S$  depends on crystal structure and temperature
  - Good thermoelectrics:  $S \approx 100$ 's  $\mu\text{V}/\text{K}$
- Ideal thermoelectric has maximum electrical conductivity and zero thermal conductivity: electron crystal, phonon glass
- Thermocouples are thermoelectric devices

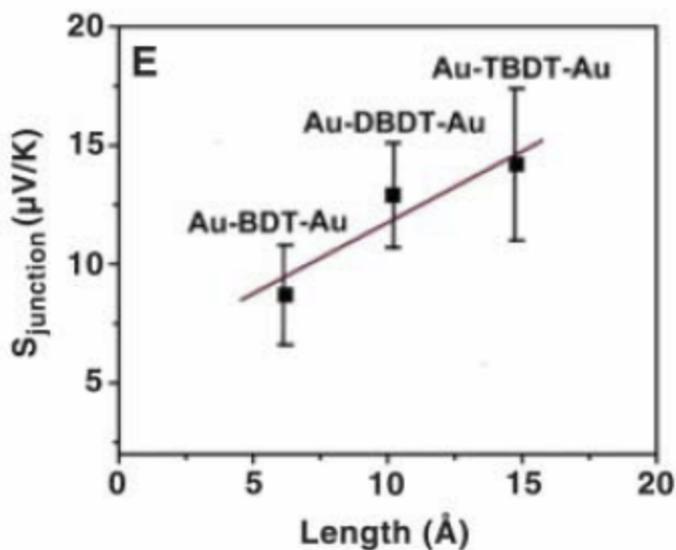
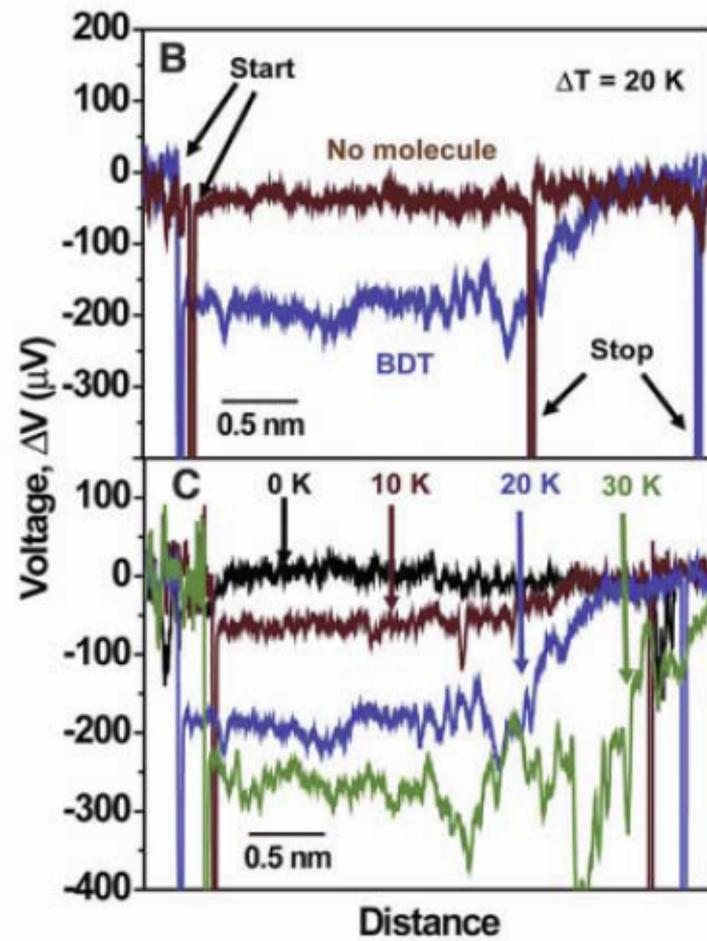
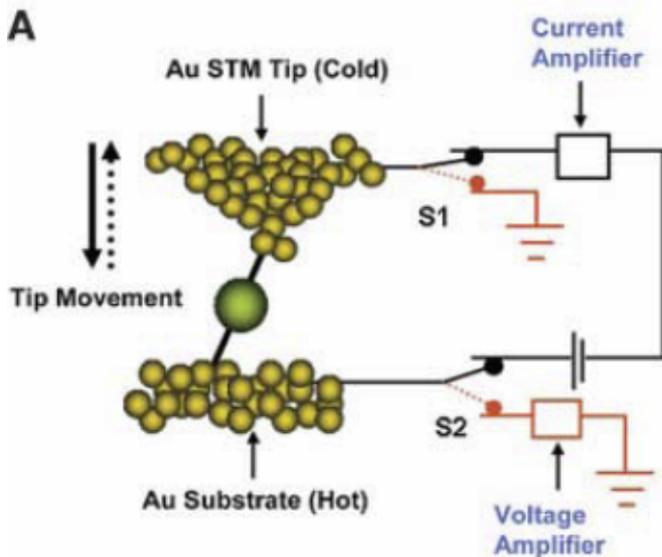
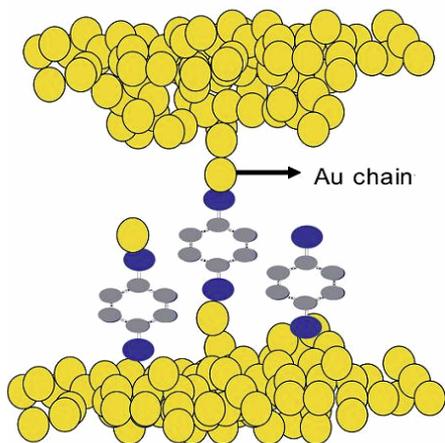
# Progress in thermoelectrics: nanoscale heat conduction effects



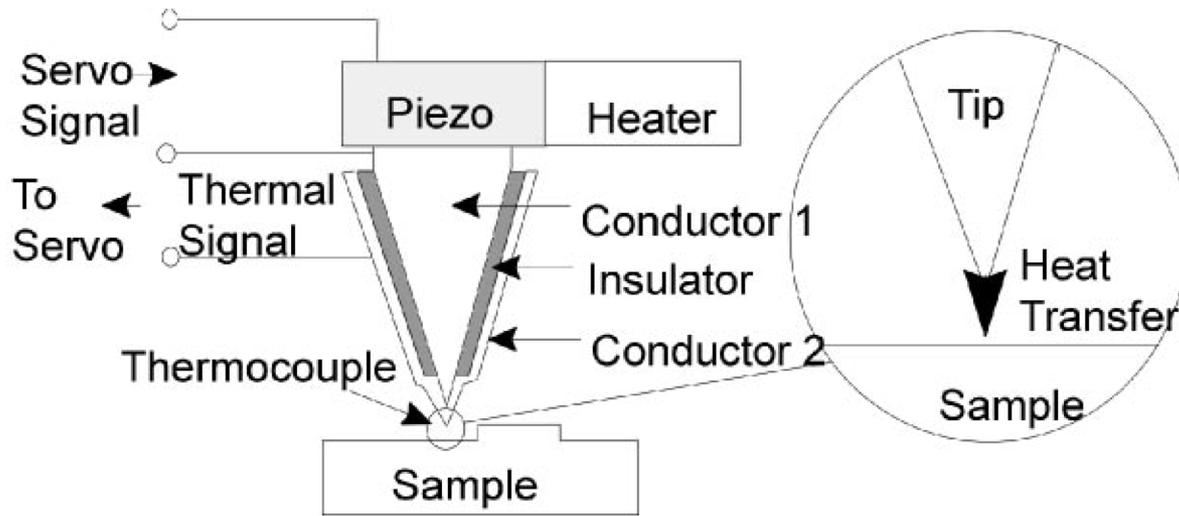
# Thermoelectricity in molecular junctions



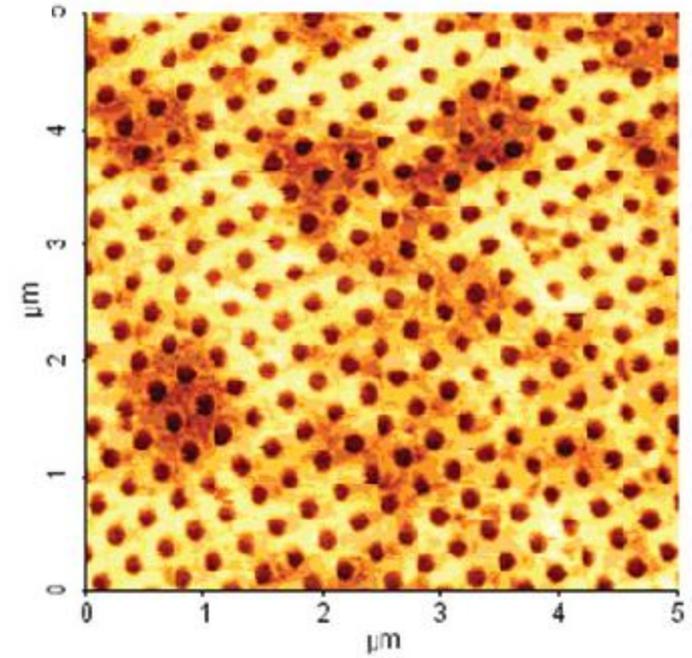
Single molecule junction, just before the junction breaks



# Scanning thermal microscopy (SThM)



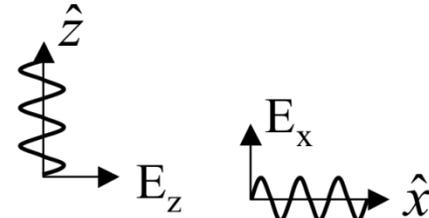
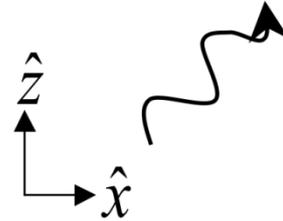
- Thermometer,  $\sim 100\text{nm}$  spatial resolution
  - Localized heat source
- Thermal conductivity map



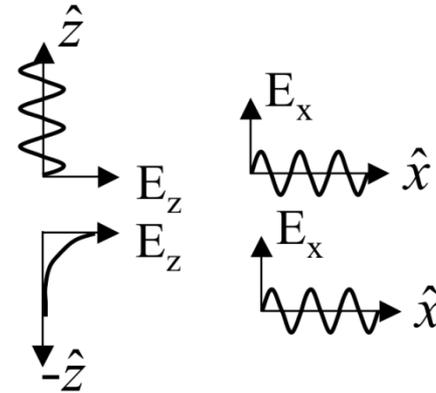
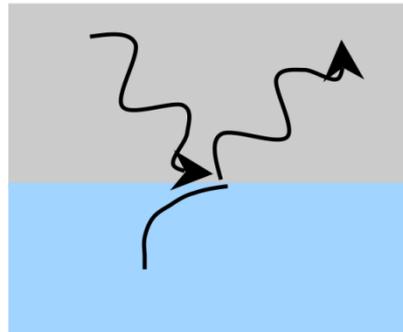
# Near field thermal radiation



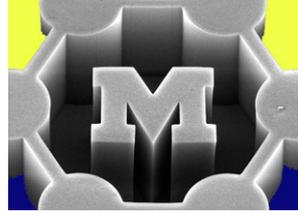
Propagating waves



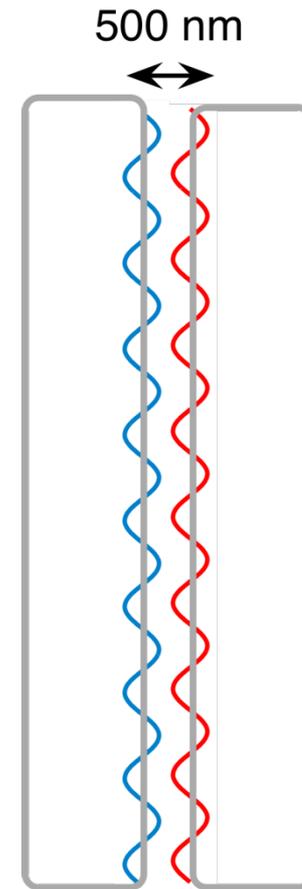
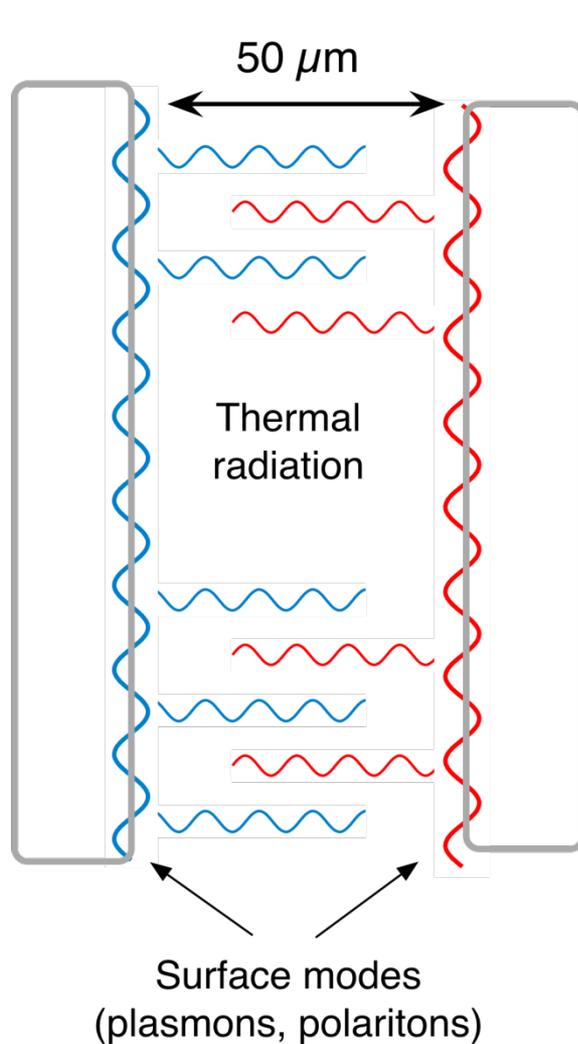
Evanescent waves:  
Decay exponentially



# Near field thermal radiation II



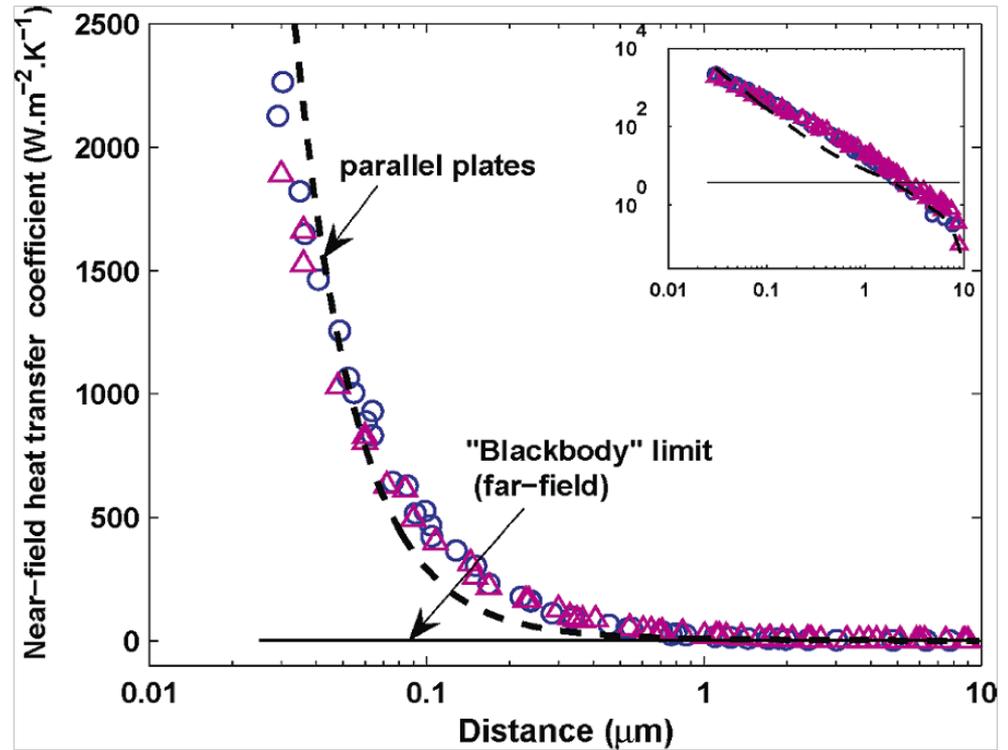
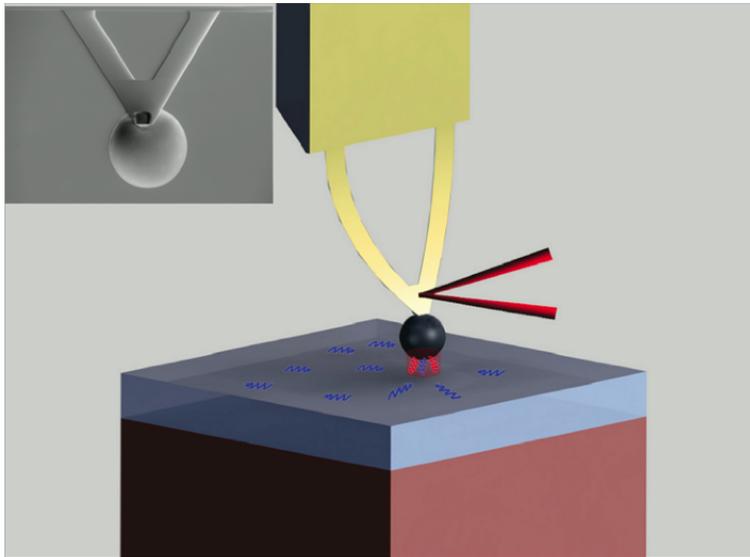
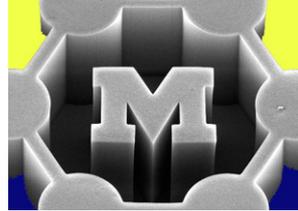
- When two objects are closer than photon wavelength, thermal radiation heat transfer can exceed the blackbody limit



Surface modes are close enough to exchange energy

# Example

- Thermal radiation between a microsphere of  $\text{SiO}_2$  and a flat surface



# Conclusion: recap of energy carriers

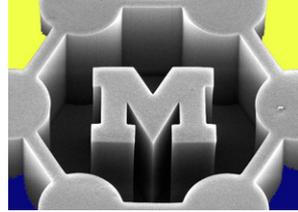


Table 1.3 Basic characteristics of energy carriers

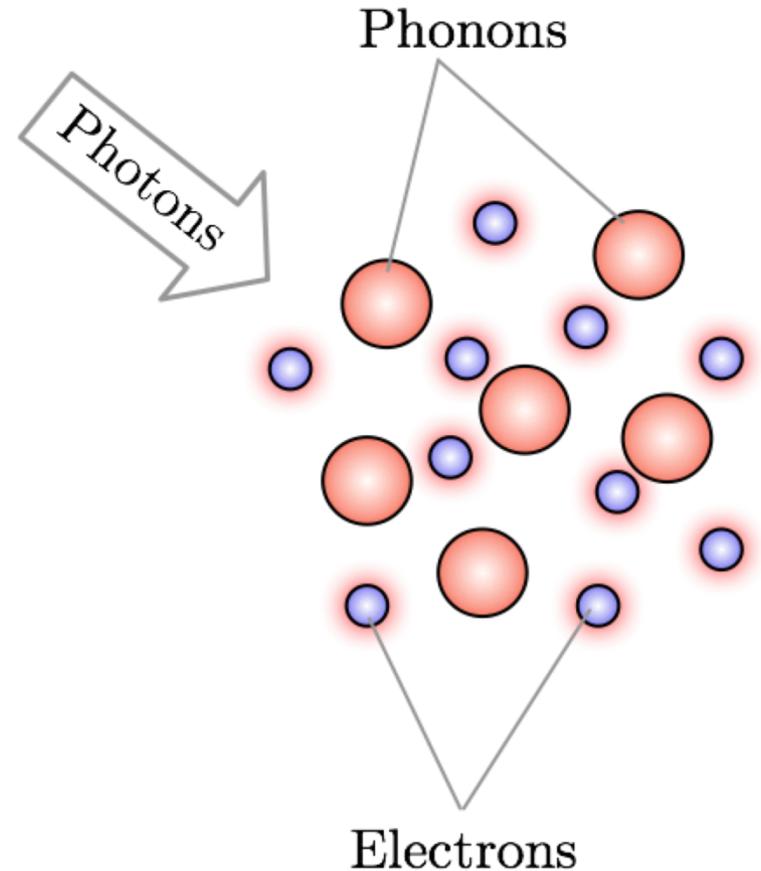
	Free Electrons	Phonons	Photons	Molecules
Source	Freed from nucleic bonding	Lattice vibration	Electron and atom motion	Atoms
Propagation media	In vacuum or media	In media	In vacuum or media	In vacuum or media
Statistics	Fermi–Dirac	Bose–Einstein	Bose–Einstein	Boltzmann
Frequency or energy range	0–infinite	Debye cutoff	0–infinite	0–infinite
Velocity (m/s)	$\sim 10^6$	$\sim 10^3$	$\sim 10^8$	$\sim 10^2$

# Non-equilibrium (fast) transport

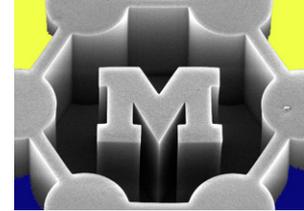
## example: ultrafast laser interactions



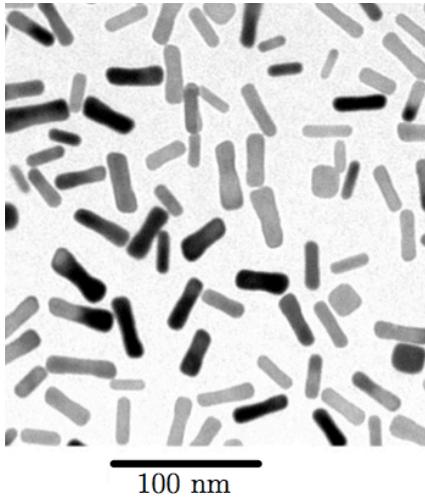
- Photon-electron  $\sim$  pulse width
- Electron-electron  $\sim$  100-500 fs
- Electron-phonon  $\sim$  1-10 ps
- Ballistic phonon  $\sim$  10-100 ps
- Diffusion  $\sim$   $t > 100$  ps



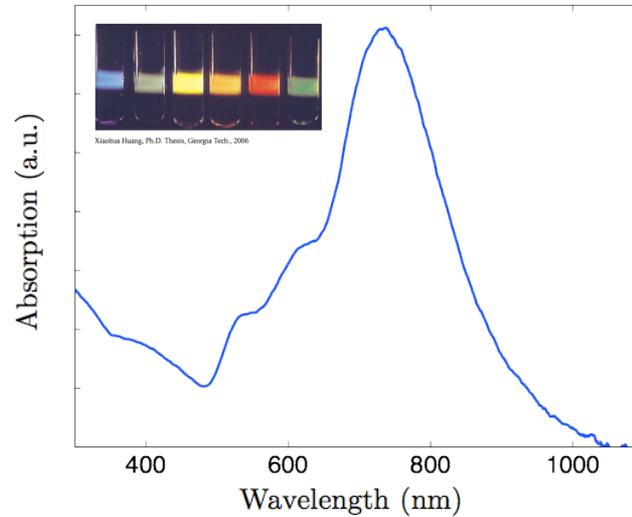
# Application: Gold nanorods for tumor hyperthermia with pulsed lasers



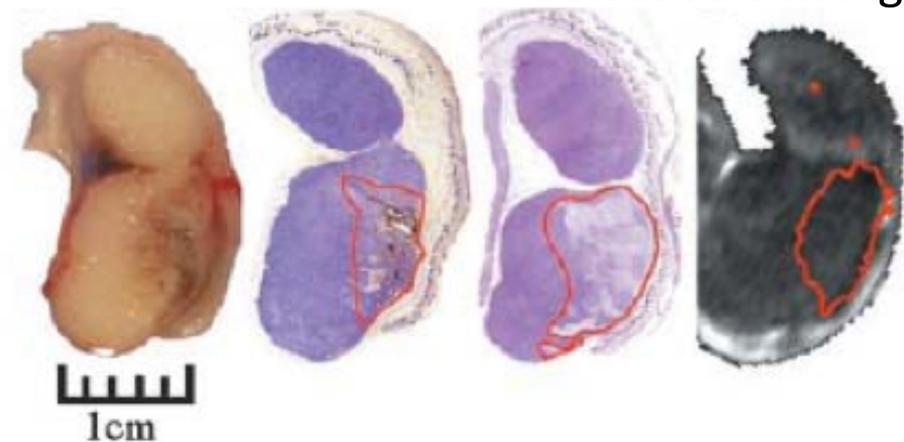
Controlled size



Tunable absorption spectra

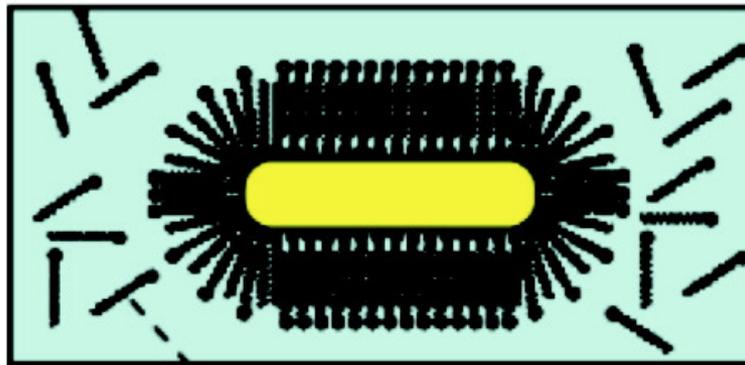
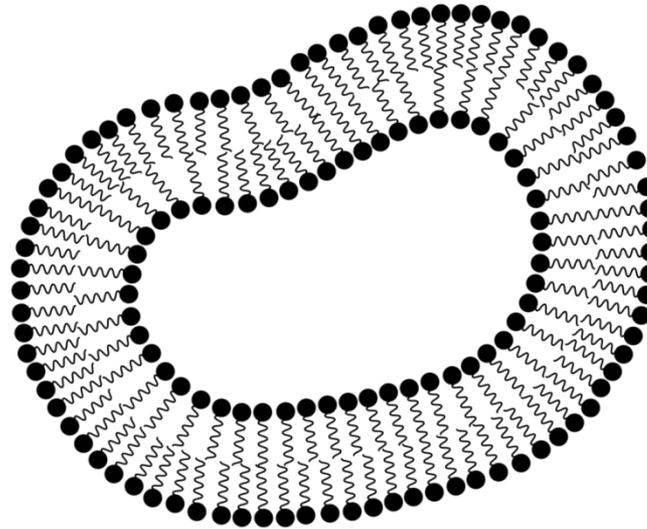
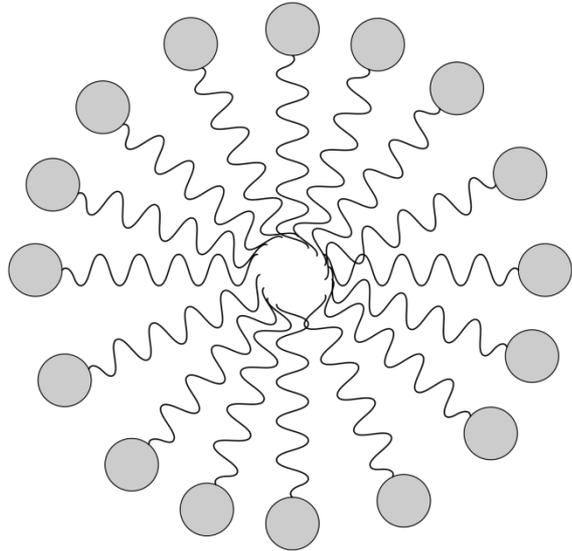
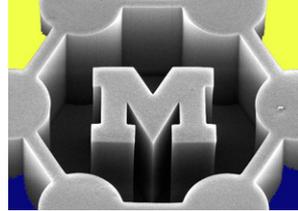


Selective binding

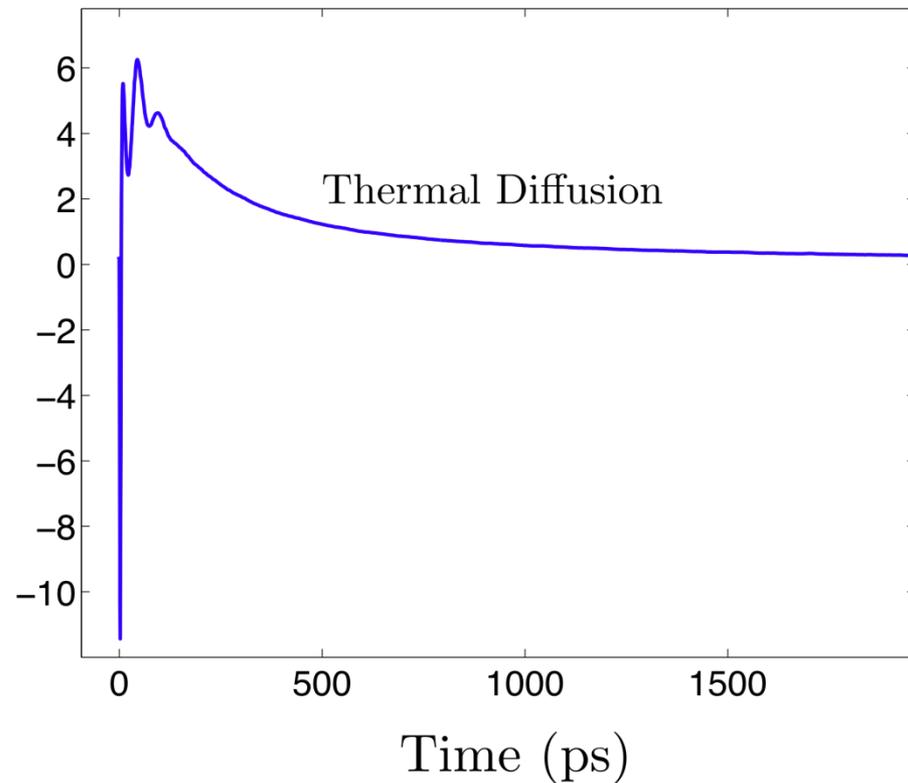
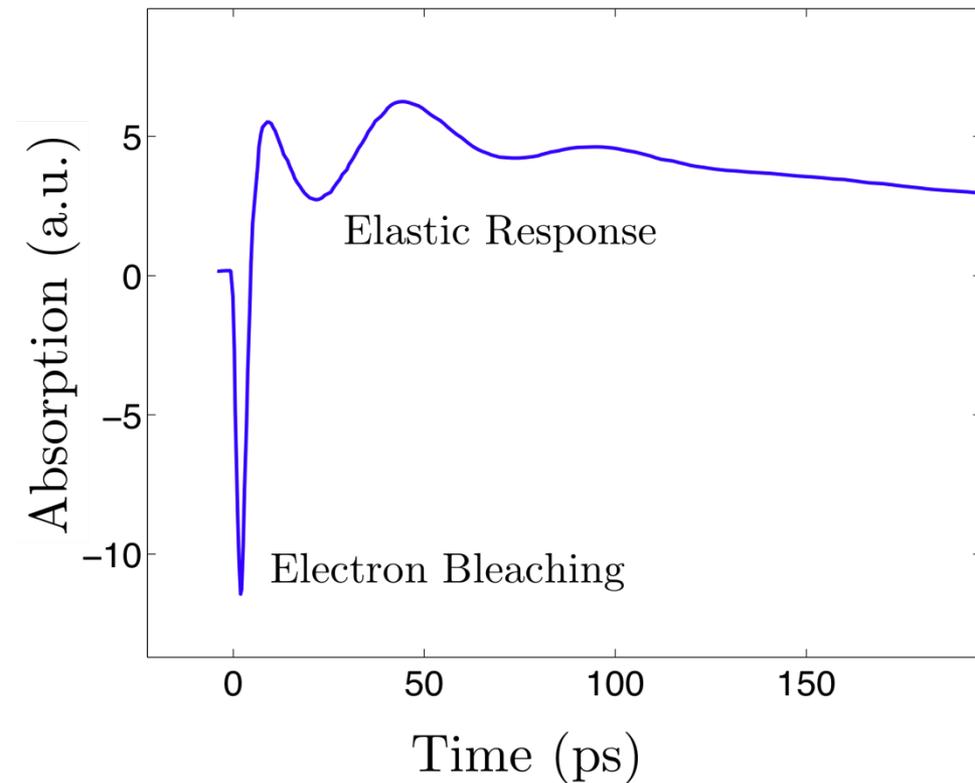
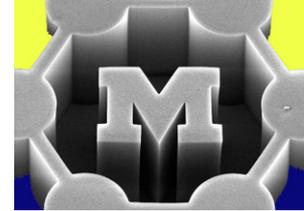


Xiaohua Huang, Ph.D. Thesis, Georgia Tech., 2006

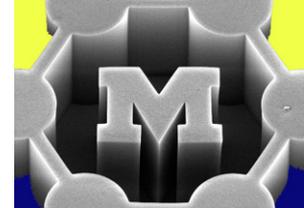
# Coating the nanorods



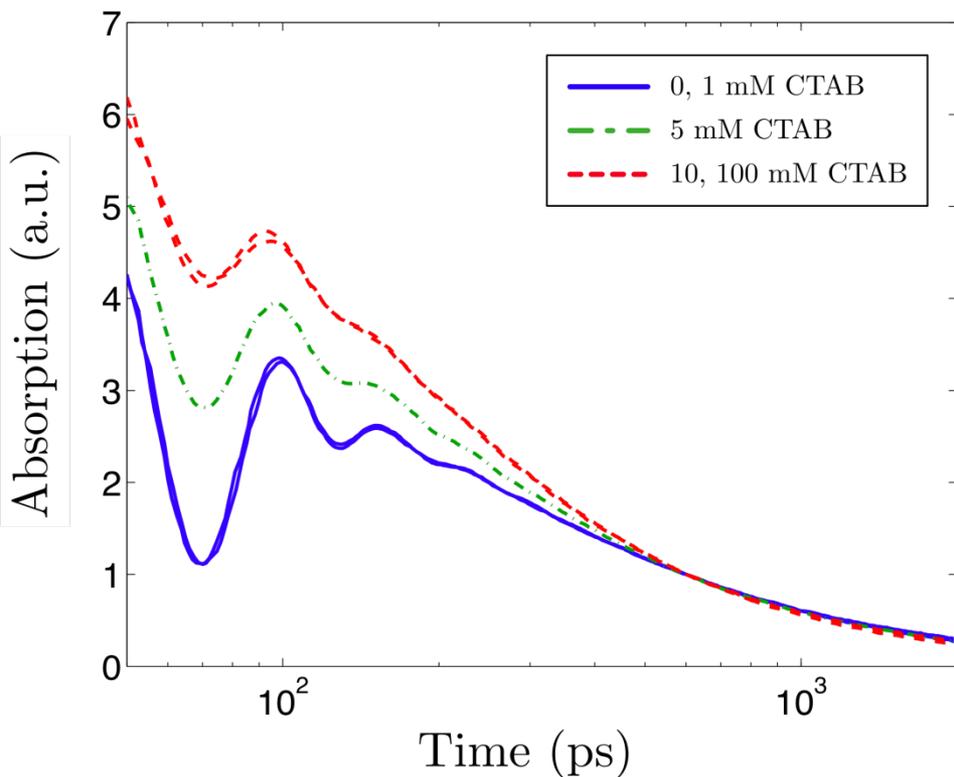
# Ultrafast absorption and cooling from laser pulse



# Measuring the effect of the bilayer



Bilayer forms with increased concentration



Bilayer decreases G

